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THE ARITHMETIC TEACHER

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The Use of Class Time in Arithmetic

Donald E. Shipp and George H. Deer

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This study* sought to determine whether varying the per cent of class time spent on developmental activities and on practice work affects achievement as measured by a three-part arithmetic achievement test. It considered the effects on achievement in Understanding Arithmetic, Using Arithmetic Accurately, Solving Problems, and Total Achievement. Also, the effects on the upper, middle, and lower ability groups were compared to determine whether these groups were affected differently by this variation in the use of class time.

The Experiment

A simple treatments-by-levels experiment, such as that described by Lindquist (1) was set up in each of grades four, five, and six. Four treatments were assigned randomly to four sections of pupils in each grade. The four sections in each grade were "matched" on mental ability at upper, middle, and lower thirds. This constituted the three levels in the experiment. The four treatments differed in the per cent of class time allotted to developmental work and to practice work. The experiment was simply duplicated in grades four, five, and six.

The study was made at the Waller Elementary School, Bossier City, Louisiana. In May, 1957, the *Henmon-Nelson Test of Mental Ability* was given to all pupils in the school who were to be promoted to the fourth, fifth, and sixth grades for the next school year. The sections for each grade were then made up by assigning at random an equal number of pupils from the upper, the middle, and the lower third of that grade according to mental ability. Four of these "matched" sections were then selected from each grade for use in the experiment.

The four treatments, whose effects were to be compared, were then assigned randomly to the four sections in each grade. Treatment A allotted 75 per cent of class time to developmental work; Treatment B, 60 per cent; Treatment C, 40 per cent; and Treatment D, 25 per cent. The remainder of the class time in each treatment was devoted to practice work. Within each grade, the same teacher taught all four sections, adjusting the activities of each section to fit its assigned treatment.

Definitions. The term "developmental activities" was used to refer to those activities of the teacher and class which were intended to increase understanding of the number system, the fundamental processes, and the general usefulness of number and quantity in everyday experience. Activities of this sort included: explanations, discussions, and demonstrations by teacher and class; han-

* A summary of this study was presented by Donald E. Shipp at the meeting of the National Council of Teachers of Mathematics at Dallas, April, 1959.

dling, inspecting, and arranging visual and manipulative materials; and group reading, drawing, construction work, and committee projects. In general, activities of the class as a group were included in developmental activities.

The term "practice work" included activities in which the pupils worked individually with pencil and paper on assigned computation or verbal problems, other exercises, and questions taken from the textbook and its accompanying workbook (2). In general, individual pupil work on assigned exercises was classified as "practice work."

Plan of the Study. One fourth-grade teacher taught the four fourth-grade sections. Each section had forty-five minutes of arithmetic class time each morning for twelve weeks. The teacher kept a daily cumulative total of the amount of class time spent on developmental activities and on practice work. Over the twelve-week period, the teacher adjusted her activities with each section so that each spent its assigned percentage of time on these two activities. In this way, the teacher was not forced into a fixed artificial division of class time each day. The teacher attempted to present a meaningful program of arithmetic, subject to the time allotments, for each section. Topics covered were those normally covered in the first twelve weeks of school. This plan was duplicated in the fifth and sixth grades.

At the beginning of the twelve-weeks period in September, the classes were given the three-part Silver-Burdett Tests, *Measuring Power in Arithmetic*. The fourth-grade sections took that form of the test covering work through the third grade. The fifth grade took the fourth grade test and the sixth grade took the fifth grade test. At the end of the twelve-weeks period the tests were again given. This time the sections in each grade took the form of the test for their present grade.

Treatment of Data. In this study there were four initial scores and four final scores for each pupil. These were the scores on the three parts of the arithmetic tests and the total score. The adjusted final test means of the sections were used to compare the achievement of the four matched sections in each grade. By making use of analysis of covariance these final test scores were adjusted in terms of initial test scores so that the four sections in each grade were statistically matched for initial ability in arithmetic.

The "F-test" was used to test for significant differences in the adjusted final means of the four sections. The five per cent level of significance was used to reject the hypothesis that no significant differences in the means existed.

When the "F-test" showed that there were significant differences in the means of the four sections, the "t-test" was applied to all possible pairs of means to locate the significant differences.

All conditions necessary for valid use of the "F-test" seem to be met in this study. Selected sets of the scores were tested for normality of distribution. Bartlett's Test for homogeneity of variance was made on several sets of scores. None of the tests was significant. Lindquist (1) states that homogeneity of variance is the most critical condition necessary for valid use of the "F-test." All sets of scores used in this study were tested for this condition. None of the tests was significant. The method of setting up the experiment and the resulting data appear to fit conditions necessary for valid use of the "F-test."

Analysis of Data

Adjusted final means for the sections in each grade were tested for significant differences by pairs in all cases where the "F-test" showed that differences were significant at the five per cent level. The adjusted final means of each section on the three parts of the test and the whole test are shown.

Summary of Data on Understanding Arithmetic. Adjusted final means for the four sections in grades four, five, and six on *Understanding Arithmetic* are given in Table I. Sec-

TABLE I

ADJUSTED FINAL MEANS FOR SECTIONS ON PART I: *Understanding Arithmetic*

	Section A*	Section B*	Section C*	Section D*
Fourth Grade	16.7	14.9	14.5	13.4
Fifth Grade	16.7	15.4	17.0	15.4
Sixth Grade	14.3	13.2	14.2	12.3

* These sections correspond to the four types of treatment described in the experiment.

tion, A, i.e., the section receiving Treatment A, ranked highest in the fourth and sixth grades and second highest in the fifth grade. Section D ranked lowest in the fourth and sixth grades and tied for lowest in the fifth grade. Data in this table appear to indicate a definite trend toward greater achievement in Understanding Arithmetic as the per cent of class time spent on developmental activities was increased.

Results of the "F-tests" applied to the data in Table I are shown below. Significant differences were found in the means

Grade	*F-test*	F_{05}
Fourth	4.24	2.71
Fifth	1.94	2.70
Sixth	3.65	2.72

of the fourth grade and sixth grade sections. The "t-test" was then made on differences in all possible pairs of means for the four sections in each of these grades. These data are not shown. Section A in the fourth grade ranked significantly higher than each of the other three sections. No other differences in this grade were significant. In the sixth grade, Section A and Section C ranked significantly higher than Section D. No other differences were significant.

Summary of Data on Using Arithmetic Accurately. Adjusted final means for the four sections in grades four, five, and six on *Using Arithmetic Accurately* are given in Table II. Section B ranked highest in the fourth grade with Section A following closely. Section D ranked lowest. In the fifth grade, Section A ranked highest with Section C lowest and Section D next to lowest. In the sixth grade, Section A ranked highest. Again, Section C ranked lowest with Section D being next to lowest.

Data showing the significance of the differences in the means are shown below. The value of the "F-ratio" at the five per cent level may be compared with the value of the "F-ratio" for the sections in each grade.

Grade	*F-test*	F_{05}
Fourth	3.86	2.71
Fifth	1.14	2.70
Sixth	3.58	2.72

Differences in the means were significant in the fourth and sixth grades. The t-test was made on the differences in all possible pairs of means in each of these two grades. Again, these data are not shown. However, in the fourth grade, Section A and B ranked significantly higher than Section D. Section B, which had the highest mean, ranked significantly higher than Section C. In the sixth grade, Sections A and B ranked significantly higher than Section C.

Summary of Data on Solving Problems. Adjusted final means for the four sections in each grade on *Solving Problems* are shown in Table III. It may be noted that differences in means in this table are very small. In the fourth grade and fifth grade, Section A ranks highest with Sections C and D rank-

TABLE II

ADJUSTED FINAL MEANS FOR SECTIONS ON PART II: *Using Arithmetic Accurately*

	Section A	Section B	Section C	Section D
Fourth Grade	30.3	30.7	28.3	27.2
Fifth Grade	26.8	25.1	24.2	24.7
Sixth Grade	26.6	26.3	23.5	24.7

TABLE III

ADJUSTED FINAL MEANS FOR SECTIONS ON PART III: *Solving Problems*

	Section A	Section B	Section C	Section D
Fourth Grade	8.08	8.00	7.96	7.96
Fifth Grade	8.04	7.68	6.42	7.06
Sixth Grade	8.12	8.68	8.51	8.62

ing lowest. In the sixth grade, Section B ranks highest with Section A lowest, although the differences are negligible.

Results of the "F-test" on differences in the means for each grade are shown below.

Grade	*F-test*	F_{05}
Fourth	.98	2.71
Fifth	2.57	2.70
Sixth	.28	2.72

None of the values of the F-test reach the value of F at the five per cent level. Therefore, data in this study do not show significant differences in the means for the sections in any grade on this part of the test.

Summary of Data on Total Score. Adjusted final means for the sections in each grade on *Total Score* are shown in Table IV. In all three grades, Section A, the section receiving Treatment A, ranked highest with Sections B, C, and D following in that order.

TABLE IV

ADJUSTED FINAL MEANS ON TOTAL SCORE

	Section A	Section B	Section C	Section D
Fourth Grade	53.8	53.6	50.4	48.6
Fifth Grade	52.1	48.2	47.2	47.0
Sixth Grade	49.0	48.1	46.1	45.6

Results of the "F-test" on differences in the means for each grade are shown below. Differences in the means for the sections in the fourth grade and fifth grade are significant. Differences in the means for the sixth grade sections are not significant at the five per cent level. The "t-test" was applied to the differences in all possible pairs of means for the fourth grade sections and the fifth grade sections. These data are not presented in this report. In the fourth grade, Sections A and B ranked significantly higher than Section D. In the fifth grade, Section A ranked significantly higher than each of the other sections. Other differences were not significant at the five per cent level.

Grade	*F-test*	F_{05}
Fourth	3.61	2.71
Fifth	3.40	2.70
Sixth	2.29	2.72

Summary of Data on Interaction of Treatments and Levels. The treatments-by-levels experimental design permits a test for interaction of treatments and levels. This is a test of the hypothesis that the effects of the treatments, whatever they are, are the same at all levels. In this study the four treatments were variations in use of class time in arithmetic, while the three levels were the upper, the middle, and the lower thirds of each class according to mental ability.

The "F-test" may be used to test the above hypothesis. Data for the "F-test" on interaction are shown in Table V. It should be noted again that the treatments were compared on the three parts of the arithmetic test and on total score. Of the twelve comparisons in the three grades, none of the tests for interaction is significant. Therefore, the hypothesis that the treatment effects are the same at all ability levels was not disproved.

TABLE V

F-TEST FOR INTERACTION EFFECTS OF TREATMENTS AND LEVELS

Grade	Part I	Part II	Part III	Total Score
Fourth	.95	.66	1.36	.57
Fifth	1.07	.34	1.86	1.82
Sixth	1.29	.73	.20	.67

$F_{05}=2.20$ for Fourth Grade
$F_{05}=2.19$ for Fifth Grade
$F_{05}=2.21$ for Sixth Grade

Summary and Conclusions

With four sets of scores for comparative purposes and with the experiment duplicated in three grades, there was a total of twelve comparisons of the effectiveness of the treatments in this study. Section A, the section in each grade receiving Treatment A, ranked first nine times, and second two times. Five times, Section A ranked significantly higher than Section D. Three times,

Section A ranked significantly higher than Section C.

Section B, the section receiving Treatment B, ranked first two times, and ranked second eight times. Twice Section B ranked significantly higher than Section D and Section C. Section C ranked first one time, and second one time. Section D ranked second one time. On no occasion did Section C or D rank significantly higher than Section A or B. On no occasion did a section rank significantly higher than another one spending more class time on developmental activities.

Data in this study seem to warrant the following conclusions:

1. There is a trend toward higher achievement, as measured by a general achievement test in arithmetic, when the per cent of class time spent on developmental activities is increased.

2. While the ideal division of class time between developmental activities and practice work could not be determined in this study, it would seem that more than 50 per cent of class time should be spent on developmental activities.

3. The above conclusions apply to all ability levels.

Further study of this problem with more refined measuring devices, especially in the area of problem solving would be of interest. Further details concerning this experiment may be found in the original study (3).

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2. Morton, Robert Lee. *Making Sure of Arithmetic.* New York: Silver Burdett Company, 1952.
3. Shipp, Donald E. *An Experimental Study of Achievement in Arithmetic and the Time Allotted to Development of Meanings and Individual Pupil Practice.* Unpublished Doctor's dissertation, Louisiana State University, Baton Rouge, May, 1958.

EDITOR'S NOTE: Here is strong experimental evidence that it pays to devote more than half of the class time to a consideration of the thinking and understanding of the developmental phases of arithmetic. There are various ways and differing methods and moments for doing this. The good teacher senses *when* and *how* to work on understanding and when it is opportune to provide practice. It is probable that no general rule should be established because some topics and some levels of pupils will require a different procedure from others. It was interesting to note that even in the area of "using arithmetic accurately" the sections that spent more time on understanding excelled those that devoted more time to practice. However, in this comparison the differences were not as large as in other comparisons. This study should cause those teachers who tend to rush into computations and practice to pause and rethink their aims and procedures.

The Little Man Who Wasn't There

Arithmetic's hero
The fat little zero
He's nothing at all, you say?

Then come with me
I want you to see
Our hero at work and at play!

With One, when he flirts
At the left of her skirts
He's done nothing at all, has he.

When he stands at her right
He's given her might—
Ten times as great is she.

Add him to One
And nothing is done
Each word that I say is true.

The same you can say
When you take him away
Its the easiest thing you can do.

But the damage was done
When he multiplied One
She completely vanished they say.

The same he would do
To a three or a two
He treats them all the same way.

Take warning and never
Try to be clever—
Division by zero don't trust.

I'm warning you now
It matters not how
Your answer will blow up and bust!

HELEN MACDONALD SIMMONS
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Defining Basic Concepts of Mathematics*

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IN THE MATHEMATICAL COMMUNITY there is a real interest in the mathematics that our children are being taught and all of us have a sincere desire to help in planning a sound mathematical curriculum. Many of you, I am sure, are aware of the several groups which unite the talents of professional mathematicians and expert teachers in an effort to make a substantial contribution to elementary and secondary school mathematics. I refer particularly to the School Mathematics Study Group under the direction of Professor E. G. Begle of Yale University, to the Commission on Mathematics of the College Entrance Examination Board, and to the University of Illinois School of Mathematics under the direction of Professor Max Beberman. I have been privileged to work with the School Mathematics Study Group and I am extremely enthusiastic and optimistic about the series of textbooks which are being tried out this year in 40 different centers. These textbooks range from the 7th to the 12th grades.

I should like to focus our attention on some of the mathematical concepts which are fundamental for work in any type of mathematics, wherever and in whatever guise it appears—whether in kindergarten, or in the pure researches of mathematicians, or in the busy engineering offices of our industries. If there are such concepts, then surely it is around these that our mathematical courses should be built.

Now when I speak of mathematical concepts I do so in distinction from mathematical techniques. Both concepts and techniques are indispensable in any mathematical situation, but I feel that in school we often lay more stress on technique than on concept. For one thing, techniques are easier to teach. One can require that a child learn that $2+3=5$, or that he know in an addition problem, where and what it is that must be carried. For another thing, techniques are easier to test; either a student can do it, or he can't. *Too often, mastery of technique passes for knowledge of the concept. On the other hand without technique no problem can be solved, no answer obtained; but without a concept, no problem can be stated and no answer interpreted.*

The concepts I should like to highlight today are two: the concept of number and the concept of a mathematical system. A mathematical system usually deals with numbers of some sort, so we need to know what a number is. But numbers, of themselves, have only limited interest. They are really interesting because of their properties, or, for what we can do with them. That is, what is the mathematical system which uses these numbers.

I want to state at the outset that I am not proposing or urging the inclusion of this material in a primary grade course. What I would urge is that the recognition of these concepts might direct the type of mathematics that is taught and even the way it is presented.

What Is a Number?

What is a *number?* What does the word "two" denote? We are often told that a number tells "how many." We often use a number as an adjective—thus we say "four sheep." From this we know only what a

* This paper was presented to Section 14 of the *Annual Conference on the Direction and Improvement of Instruction and on Child Welfare* held in Los Angeles, November 16 to 20, 1959.

number does, not what it is. "Four" can also be a noun. It is the name of a number. What *is* a number? Your students should be aware that a number is an entity in itself. Perhaps as abstract an entity as love, but an entity nevertheless.

I recall being present at an exchange between a small boy, the son of a mathematical colleague of mine, and a physicist. The father was very proud of the boy's ability in arithmetic and when the physicist offered to test the boy, the father was delighted. The physicist asked the boy, "What is 2+2?" instantly the boy replied, "Four." The look on the boy's face and his father's face clearly indicated that the boy's intelligence had been insulted. The physicist came back immediately with the question, "Four what?" The facial expression on the boy became puzzled and the father's face fell. No one answered the question and the physicist politely changed to another, more reasonable, problem. What answer would you have given? What is "four?"

As is common place in mathematics, to answer such a question, we must consider other concepts, even more basic than that of number. Perhaps a good answer to the question "four what" is "Four of anything." A lot is involved in such an answer. The phrase "of anything" suggests that we must be speaking of a collection of things. The mathematical word is the common word "set." Here is a fundamental concept from which all mathematical work must begin: concept of a set. What is so impressive about a set? It is just a collection of objects. What can you do with sets? Indeed, there are many general laws which hold for all sets. In fact, there is a whole mathematical system of sets to which some mathematicians devote their entire research, but this is another story.

Let us look at some sets.

1) The set consisting of the fingers on my right hand.
2) The set consisting of the corners at the intersection of two streets.
3) The set consisting of the games won by the Dodgers in the World Series.
4) The set consisting of the wheels on an automobile.
5) The set consisting of the examples I have just cited.
6) The set consisting of a row boat, a hydrogen atom, a potato chip, and the abstract concept of friendship.
7) The set consisting of the marks 1 1 1 1.

What do these sets have in common? Besides being listed by myself just now, all sets have the common property that they contain exactly four members. While this is true, you may well ask how can I use these examples to tell you what four is. I certainly cannot suggest a definition of four which uses the word "four." I need, therefore, still another basic concept to explain "fourness." Let us look at two of the examples above. The set consisting of the fingers on my right hand and the set consisting of the corners at the intersection of two streets. I can, if I so desire, match each of my fingers with one of the street corners. Take any two of the sets cited above. I can match up the members of one set with the members of the other, so that each member of the one set is matched with one and only one member of the other set, and conversely. Such a matching is in mathematics called a one-to-one correspondence. Now I think I am ready to admit to you what I consider the number four to be. It is the abstract property possessed by all sets that can be matched, placed in one-to-one correspondence, with the fingers of my right hand. Of course, I could have replaced the fingers of my right hand by any set I might care to designate as long as it could be placed in one-to-one correspondence with the fingers of my right hand. I could have used the set of marks, 1 1 1 1. Indeed it is just such a set that is used as the prototype for the number 1, and the reason that so many civilizations have used a symbol resembling a single stroke to denote one.

Again I want to emphasize that I am not suggesting that this is the way the number concept should or even could be presented to primary graders. I do suggest however,

that the concept of set and the word set is an idea and a term which can and should be introduced. Its use can make your language more precise. For example, when you ask the first row of a class to rise, you really mean the set of students sitting in the first row. The work books I have seen for the first and second grades are filled with different sets and many questions are asked about these sets, although the set concept is not mentioned. Introduction of the term would unify these different problems. Moreover, emphasizing the set concept associated with "four" enables us to think of four as associated with a set of dissimilar objects. Indeed, why must the concept of four always be illustrated by four identical objects? Why not a collection of four different items? There is as much fourness in such an example as in the example of four apples.

The concept of matching, or one-to-one correspondence, is a concept which can be introduced at a tender age. For example, what is the easiest way of being sure that each child in your class gets his bottle of milk? Counting the students and then counting the milk bottles is a sophisticated way, and it is the source for much error. However, if you were to send each child to pick up his own bottle, you would have a sure-fire method because a one-to-one correspondence between the set of pupils and their milk bottles has been established physically.

I should like to end our discussion of the concepts of set and one-to-one correspondence by posing a short problem which illustrates the power of a one-to-one concept.

If we estimate the number of people in this room as being in excess of 31 people then we can assert without fear of contradiction that at least two people here must have their birthdays on the same day of the month. How can we be so sure? Suppose you were to try to make a one-to-one correspondence between the people at our meeting and the days of a 31 day month. After you have matched 31 persons with a day in the month, say by associating each person with his own birthday, you would have people left over. Now at this point either two people have the same birthday, in which case our assertion has been established, or all the days of the month have been selected. However, you have people left over; the next person then must have a birthday which will coincide with one of the first 31. Here the essential idea is that of a one-to-one correspondence. I might add that this is one of the problems in the experimental text now in use in 7th grade and it has the parents busy. The children do fine with it.

Thus we have seen that the number concept entails the concept of set and the concept of a one-to-one correspondence. These concepts are more general than numbers, and they are more abstract. Certainly some students will make the abstraction more easily than others and apply the principle more widely than others. These are some of the individual differences this section is discussing. I feel sure that you will find that some students who have been slow to master computational techniques will grasp these general abstract ideas more quickly than their fellow students. Of course, the gifted student is good in anything and a student less than average will have difficulty with anything.

This has been a brief discussion about what I like to call the counting numbers. Now we can ask, what can we do with them, what good are they? What we can do with them is given by the properties of the mathematical system to which they belong. For the purposes of this talk, I should like to say that a set of numbers becomes a mathematical system when operations are defined on the set and the laws these operations obey are listed. For the set of counting numbers the operations are addition and multiplication. What does an operation do? It says that if we combine two elements we obtain another element of our set. Thus addition means the combining of two numbers so that a third number is obtained. A similar remark is true about multiplication. What are the laws they obey? They obey the law that $2+3=5$ and $2\times3=6$. Also a lot more. In fact they obey all the rules of the addition

and multiplication tables. But I should like to single out some of the general laws these operations obey.

The Commutative, Associative, and Distributive Laws

THE COMMUTATIVE LAW: $a+b=b+a$ and $a\times b=b\times a$.

Why is it that $2+3=3+2$? Let us interpret addition physically. Let 2 be associated with 2 sheep and 3 be associated with 3 balls. If we are asked for the number of objects in the set consisting of 2 sheep and 3 balls, that number is 5. It is clear that this interpretation of addition yields a commutative operation. Please notice that I have just added in a set theoretic fashion quantities which are unlike. However, if I regard addition as an operation which has just been defined for the counting numbers, then I have to write down the commutative law as an axiom which is to be obeyed. That is, I must assume it.

I will demonstrate that $a+b$ need not always be equal to $b+a$ by considering another mathematical system. The set of elements of my system are the various actions which people perform. The operation $a+b$ is to mean: "first perform action a, and then perform action b." For example, if a is the act of putting on a coat and b is the act of putting on a hat, then $a+b$ is the act of putting on first a coat and then a hat. And it is clear that $a+b=b+a$. If however a is the act of putting on a stocking and b is the act of putting on a shoe, then it is clear that $a+b$ does not equal $b+a$. The stocking had better be put on first!

Other properties which hold are the Associative Laws:

$$a+(b+c)=(a+b)+c$$

and

$$a\times(b\times c)=(a\times b)\times c.$$

Our operations were defined so that they combine numbers two at a time. Thus when confronted by $1+2+3$, I have to combine either $1+2$ first or $2+3$ first, and then add the third number. Why is it that you always get the same number? If you interpret these operations in terms of sets, the reason is clear. If you think of this from the point of view that we are constructing a mathematical system then we must take the associative law as an axiom.

Another law which you may take as an axiom is the DISTRIBUTIVE LAW:

$$a\times(b+c)=(a\times b)+(a\times c)$$

In conjunction with the commutative law, this can also be written

$$(b+c)\times a=(b\times a)+(c\times a)$$

To illustrate the importance of these laws we need only look at some of the standard arithmetic techniques. Applications of the associative law assure us that if we add a column of figures up, we get the same answer as if we had added them down. Writing $16+7+4+3$, I think will point up the need for the associative law. Moreover, if you combine them $(16+4)+(7+3)$ in order to obtain an easy addition, then you have invoked both associativity and commutativity. The distributive law, together with the associative law, gives us the "carry" rules of multiplication. Thus

$$\begin{aligned}16\times 7 &= (10+6)\times 7 \overset{D}{=} (10\times 7)+(6\times 7)^{*}\\ &= (10\times 7)+42=(10\times 7)+(40+2)\\ &\overset{A}{=} ((10\times 7)+40)+2=((10\times 7)\\ &\qquad +(10\times 4))+2\\ &\overset{D}{=} (10\times(7+4))+2\\ &\qquad =(10\times(10+1))+2\\ &\overset{D}{=} ((10\times 10)+(10\times 1))+2=1\times(100)\\ &\qquad +1\times(10)+2\times(1)=112.\end{aligned}$$

Now again, let me remind you, that I am not suggesting this description of the mathematical system formed by the counting numbers is the one to be presented to students in

* Here a "D" or an "A" above an equality indicates that the equality holds because of the distributive law or the associative law respectively.

primary grades. There are other beautiful ways to motivate these algebraic rules. What I do urge you to present is the recognition and the scope and the power of these general laws.

Moreover, I shall risk my popularity by asserting that any teacher should know more sophisticated reasons for a rule than the one he teaches his pupil. He should be aware of alternative descriptions for the usual procedure. One of the reasons I feel strongly about this is to take care of "individual differences." A teacher must be ready to adjust his own way of doing a problem to the suggestions of his pupils. Individual differences appear in the approach taken toward a particular problem, or the understanding given a concept. A teacher must be flexible enough to adapt varied approaches and encourage imagination and originality even if they do not coincide with what the book says. I do not see how he can do this if he has learned his subject from his own text.

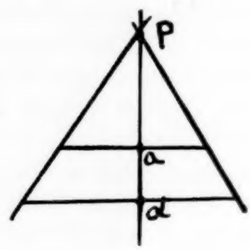
Geometric Concepts

Other concepts which can be introduced early are those of a geometrical nature. This is a time when we are acutely aware of space. Spacial terminology is present in much of the material that meets children on television and in other media. Spacial perception and the geometrical properties of space can be introduced early in school. Professors Suppes and Hawley of Stanford University have taught a modified version of much of Euclid's Elements, Book 1, to first grade students in Burlingame. They found that points, lines, and ruler-and-compass constructions can be made meaningful by placing rulers and compasses in the hands of the youngsters.

Recognition of geometric figures, both planar and spacial, are important. Also important are incidence relations: points lie on a line, lines go through points, the intersection of lines and the intersections of planes. I think primary graders can draw curves and closed figures and can be taught what inside and outside are. Many of these geometric concepts can be linked with number concepts. Thus a triangle means 3, a rectangle, 4, a pentagon, 5, and a hexagon, 6.

Here too is a good opportunity to use such concepts as set and one-to-one correspondence and to point out the connection between the various parts of mathematics. Thus a line is a set of points and a point is the intersection of two lines. A circle of radius r with center p is the set of points, each of which is at a distance r from the point p. Notice I have replaced the usual word locus with the word set.

Let us take a short line and a long line. Are there as many points on the short line as on the long line? What about a one-to-one correspondence? In the figure below the



endpoints have been joined to form two lines meeting at p. Then lines through p determine matched points, a and a' on the two lines. There are many such relations between points and lines which do not require much of Euclid, but the concepts are important. Permitting the child to make these constructions will permit him to discover the laws himself.

Recognizing Special Aptitude

One of the questions before this section of the conference concerns individual differences. How can you recognize the signs of special aptitude for mathematics? My own criteria run along two main lines.

First, does the student have original ideas? Does he invent new ways of solving problems? Can he provide equivalent but different answers to the same problem? Does he invent his own shortcuts? Thus if I found a student who multiplied as follows:

$$\begin{array}{r} 16 \\ \underline{37} \\ 42 \\ \\ 70 \\ 180 \\ \underline{300} \\ 592 \end{array}$$

or better yet

$$\begin{aligned} 16\times 37 &= 16\times(40-3) \\ &= 640-48=592, \end{aligned}$$

I should like to see further instances of his creativity at work. I think a good teacher will encourage originality and resist the temptation of demanding uniform answers.

Second, I would watch for a student to generalize from a few examples to a general rule. Combined with the ability to abstract what is really essential in the solution of a problem, this represents sophisticated mathematics. A specific problem has no value, but the principle or concept used may be of great value. That is why I lay stress on recognizing the general rules which govern mathematics. Moreover, I believe that a student who understands the why and wherefore will more quickly and more thoroughly learn the conventional skills. Such a student will be able to use his mathematics in new situations not covered by specific examples.

We have a real opportunity and a responsibility to make substantial improvements in our curriculum. No longer will it suffice to consider merely the social aspects of mathematics. It is necessary for us to consider those aspects of mathematics which will increase the intellectual level of our students, which will pique their native intellectual curiosity and reward them for time spent in intellectual pursuits.

EDITOR'S NOTE. It is apparent that elementary school mathematics will be entertaining some new ideas which will lift it above the level of skills and techniques. Just how and where these ideas will be used will be the subject of experiment for a number of years. One can foresee a fruitful cooperation of mathematicians who in years past were little concerned with the elementary school and teachers whose minds are open to new ideas. It is expected that The National Council of Teachers of Mathematics may provide the organization that will bring together the best thinking of the several groups that should be concerned with the program of arithmetic and mathematics for our elementary schools. Perhaps it is timely to repeat that our schools exist so that youngsters may learn those things that are important to them and to the society in which they live. Value judgments must be made. Who shall make these judgments? If the decision is given to the "experts," then who will choose the experts? If the future shall be determined by the "rank and file" will progress be made? Many serious problems lie ahead. They will not be solved by being ignored.

A One-Handed Clock

In teaching kindergarten or first-grade children to tell time, it is helpful to begin with a clock which has only one hand, the hour-hand. Children are ready to use such a clock as soon as they recognize the numerals 1 to 12. They can associate the time at which the hand points to 9, as time for school; the time at which it points to 12, as time for lunch. Similarly other critical hours of their day can be pointed out on the clock.

Movement of the hand on a cardboard clock to specific positions should be accompanied by such language as:

The clock shows nine o'clock.

Now it is between nine o'clock and ten o'clock.

It is a little after nine o'clock.

It is about halfway between nine o'clock and ten o'clock. We call this half past nine.

It is closer to ten o'clock than to nine o'clock.

It is almost ten o'clock.

Now it is ten o'clock.

Experience indicates that the one-handed clock is entirely adequate and satisfying in meeting the needs of the young child for time-telling, and it builds readiness for more precise time-telling with the standard two-handed clock.

CAROLINE HATTON CLARK

The Vocabularies of Five Recent Third Grade Arithmetic Textbooks*

Florence C. Repp
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Specialists in both reading and arithmetic recognize the importance that vocabulary plays in the readability of arithmetic textbooks. Studies dealing with different aspects of vocabulary load have contributed to the improvement of the reading matter in arithmetic texts. Teachers are, however, sometimes at a loss as to how to compare the respective vocabularies of different arithmetic books since there has seemed to be no common basis for comparison. The present study attempted to arrive at a common basis for comparison of the vocabularies of several recent arithmetic textbooks.

Statement of the Problem

The present study tabulated and compared the vocabularies of five widely used arithmetic textbooks of third-grade level published during the period of 1952 to June, 1955. To assure a fair basis of comparison, all words and their variants were counted as separate words.

Several supplementary problems arose. These were:

(1) To find the actual words used and the number of different words for each book.
(2) To find the average number of new words per page for each book.
(3) To find the exact number of new words for each page for each book.
(4) To find the number of books in which each word occurred.
(5) To find which words and how many words were found in only one book, in two books, in three books, in four books, or in all five books.

To be most usable, the reading material in arithmetic textbooks should not afford the child undue difficulty. Textbook writers and publishers have been aware for some time of the need for a controlled vocabulary. Much work has been done along this line in the field of reading. Similar studies in arithmetic vocabulary have lagged behind. One problem in vocabulary study is to find a common basis on which to compare and tabulate vocabularies of currently used arithmetic textbooks. The vocabularies of primary grade texts as listed in the teacher's guides or in the books themselves have been based on such varied assumptions that a common ground for comparison had to be established. Therefore, this study tabulated and compared the *actual words* in the five vocabularies *on the same basis*.

Limitations of the Study

This study admittedly did not deal with reading difficulty of arithmetic textbooks as a totality. It did not deal, certainly, with the difficulty of ideas or mathematical content. It did not deal with children's difficulty in *learning* the words used in the books, a very important factor to be considered in judging reading difficulty of text material. It did not consider sentence length or sentence structure or any of the other related factors which are considered in one or another formula for judging reading difficulty. Omission of these considerations is not intended as an indication that they are of subordinate impor-

* Florence C. Repp, "A Study and Comparison of the Vocabularies of Five Recent Third Grade Arithmetic Textbooks" (Unpublished Master's Thesis, University of Alabama, 1956).

tance. Detailed study of the texts was possible (because of time limitations) on only one factor; the writer chose quantitative load of vocabulary as one of the important difficulty factors and concentrated on it. The publication date limits of 1952 to June, 1955 were selected because major publishers of several widely used arithmetic textbooks had revised their texts since 1951 and the new editions were available. June, 1955 was the date this study began. The particular five books were selected because they were representative of modern arithmetic texts used by a majority of children.

Tabulation of Words

To remove all doubt concerning variant forms of words, all forms of all words were counted separately. The specific manner of dealing with particular forms is indicated below:

Capitalization did not affect a word.
(*Miss Hall* recorded as *miss* and *hall.*)

In hyphenated words each part of the compound word was counted as a separate word.
(*ice-cream* recorded as *ice, cream*)

Contractions were counted as new words.
(*I'll, we'll*)

Possessives were counted as separate words.
(*ones, one's, ones'*)

Abbreviations were counted as separate words.
(*A.D., A.M., yd., in.*)

All proper names were counted as separate words.
(*Tom, James, Susan, Helen*)

Purposely mispelled words were counted as separate words.

("Say hundred not hunderd." *Hunderd* counted as a word.)

The plurals of all nouns were counted as separate words whether they were regular or irregular.
(*girl, girls; foot, feet; calf, calves*)

All variants of the verbs, whether regular or irregular, were counted as separate words.
(*play, plays, playing, played; go, goes, went, gone*)

The comparative and superlative forms of adjectives and adverbs, whether regular or irregular, were counted as separate words.
(*tall, taller, tallest; well, better, best*)

All different words in each book were tabulated according to the page on which the word first occurred. The words appearing in illustrations were counted as well as the words in the text.

A master word list was compiled on 3329 file cards, arranged in alphabetical order, showing the actual vocabulary of the five books. The cards furnished three pieces of information; the word, how many books it appeared in, and which books they were.

Definition of Terms

Each word as it appeared for the first time in a particular book was called a "new word."

"Pages of text" include the frontispiece through the last page the children are supposed to read. Tables for practice, indices, and information for the teacher are not included in that number.

"Grand total" is a summary of all the different new words appearing in a particular book.

Results

Because of the length of the table listing the words used in all books, it cannot be reproduced here. The very bulk of this table impresses the reader with the size of the vocabulary load (quantitatively) of the aggregate number of words in the five third-grade arithmetic textbooks examined.

The total of all the new words appearing in any one particular book is shown in Table 1 under the heading of "Grand Total." Several interesting relationships appear in this summary table. Book A has 2096 different words, which is one and a half times as many words as Book B with 1379 words. Books D and E have almost exactly the same number of words, although Book D has seventy more pages than Book E. In comparing the two books that have the largest number of words, Book A has 107 more words than Book C.

The average number of new words per page is smallest for Book D (3.98) whereas it is largest for Book A (6.78). In fact, Book A approaches having twice as large an average as Book D. Book B has, on an average, almost 3 new words per page less than Book A. Book C, with the second largest number

TABLE 1

SUMMARY OF VOCABULARY TOTALS AND AVERAGE NEW WORDS PER PAGE

Book	A	B	C	D	E
Grant Total	2096	1379	1989	1498	1499
Total pages of text in each book	309	328	326	376	306
Average number of new words per page	6.78	4.20	6.10	3.98	4.89

of words, has also the second largest average number of new words per page. Book B, which has the smallest number of different words, has next to the largest average number of new words per page. This "average" number of words per page is the number of words that would appear on each page if the total number of new words for the book were to be distributed equally on each of the pages of that book.

While the average number of new words per page is of some importance in judging the vocabulary load placed on the child reading the book, it does not indicate the wide range of the quantitative vocabulary load, page by page. The actual number of *different new words, page by page*, adds much to the total picture, for it tells how well the introduction of new words is distributed throughout the book. This information was summarized in detail in the original report of this research; only a condensed version of the results is possible in this article. (See Table 2.)

All the textbooks have many pages with no new words as shown in the bottom line of Table 2. The number of such pages in any one book range from 43 to 104. A total of 348 pages out of 1645 pages have no new words on them.

At the other extreme one should consider the *largest number of different words* appearing on any one page in each book. Four of the books have pages on which the reader meets more than 60 new, different words on a single page! Book A has a page with 62; Book C, 63; and Book B, 64. Book E has the largest number, which is 69 different new words on one page, while Book D has no page with more than 49. The heaviest vocabulary load for any single page of Books E and D differs by twenty words.

Since the average number of new words per page ranged between 4 and 7 words (see Table 1), an examination of actual pages of text within that range may be useful. Table 3 presents this information for this limited range, giving more detail than was possible in Table 2. In Book A, 32 pages have 4 words on each of them, 18 pages have 5 words, another 18 pages have 6 words, and 16 pages have 7 words. This makes a total of 84 pages

TABLE 2

DISTRIBUTION OF NUMBERS OF PAGES HAVING CERTAIN NUMBERS OF *New Words* ON A PAGE

No. of Words	Book A	Book B	Book C	Book D	Book E	Totals	Cumulative Totals
61–70	1	1	1		1	4	4
51–60	2					2	6
41–50			2	1		3	9
31–40	6	2	5	3	4	20	29
21–30	12	5	8	3	3	31	60
11–20	40	25	46	29	25	165	225
1–10	205	219	195	236	217	1072	1297
0	43	76	69	104	56	348	1645
Total	309	328	326	376	306	1645	1645

TABLE 3

DETAILED DISTRIBUTION OF PAGES HAVING "AVERAGE" LOADS OF NEW VOCABULARY

No. of Words	Book A	Book B	Book C	Book D	Book E	Totals for Detail	Cumulative for Study
7	16	11	13	11	8	59	421
6	18	19	18	8	16	79	500
5	18	17	12	25	25	97	597
4	32	35	25	24	26	142	739
Totals	84	82	68	68	75	377	

for Book A in the range of 4 to 7 new words per page. Book B has 35, 17, 19 and 11 pages within that range, with a total of 82 pages. In the same way we find Book C has 68 pages, Book D also has 68 pages, and Book E has 75 pages with a word load roughly equivalent to "average load." The totals of 377 different pages within this range are less than a fourth of the total pages. In other words, more than three-fourths of the pages in all books examined are outside the range of averages. This shows how limited the "average" vocabulary statistics may be when one seeks to find out the real vocabulary load in day by day use of a book.

Table 4 shows that the aggregate number of different words found in the five books is 3329. About two-fifths of this total (1327 words) are used in only one of the five books. The total words used in two books, three books, and four books is 1304, or only twenty-three words less than the number used in only one of the five books. Six hundred ninety-eight words (about one-fifth) of the aggregate words were used in all five books. The words occurring in only one book (1327) are just 52 short of being as many as the entire different, new vocabulary of Book B (1379).

Some Interpretations of the Results

The writer was impressed by the aggregate number of different words used in the five third-grade arithmetic textbooks. Three thousand, three hundred, twenty-nine words are included in the combined vocabularies of five books which are used in one third-grade content subject. The wide variation in the number of different, new words for each text is not evident in a cursory examination of the book, but this study showed the two extremes to be 1379 and 2096.

The uneven distribution of the quantitative vocabulary load seems inconsistent with the many other evidences of constructive planning of the texts. There were 500 pages in the five texts with 6 or more different, new words on them. Seven pages had 49 or more words on them. The number of new words which were used in only one of the five books lacked 52 words of equaling the size of the

TABLE 4

TOTAL OF WORDS ACCORDING TO NUMBER OF TEXTS IN WHICH THEY APPEAR

Number of Books Using a Word	Number of Words
Five books	698
Four books	335
Three books	366
Two books	603
One book	1327
Total	3329

different, new vocabulary of one of the texts! Some of these words are doubtless more effective for the book in which they were used than any other word would have been; nevertheless, it seems that this number of words is unnecessarily large, and might afford transient children difficulty with a vocabulary they had not recently reviewed.

A large proportion of the "words" for the present study are variants of a root form (not counted by publishers in making statements about vocabulary load). A check of the first 7 pages of the master word list shows four abbreviations, 27 regular plurals, 7 regular possessives, 20 proper nouns, and 16 regular verb variants or a total of 74 words (out of 168) which would not have been listed in the typical vocabulary study. To exclude these would represent a 44 per cent reduction in the word list for these pages. It is hardly safe to assume that children learn all word variants even if they know root forms.

The classroom teacher needs to be aware of the different new words in the texts she uses with children. This is frequently difficult to ascertain, since publishers do not give a list of different, new words for each text, but often base the vocabulary, when it is given, on a certain series of reading textbooks. With vocabularies of different texts ranging from 1379 to 2096 different words, the child may find the vocabulary load beyond his capacity. A child having reading difficulties may well become discouraged with arithmetic when he encounters pages of written matter which introduce 62, 63, or 64 different words new to that book on a single page.

It becomes important for the teacher to notice which pages in a text present a heavier vocabulary load than the others. The 225 pages in this study that have more than ten words new to a page may require the teacher to do more teaching on vocabulary. These pages may not be as well adapted to independent study for the pupil.

Especially in schools where there is a shifting enrollment, care should be taken by the teacher to check on the vocabulary of the previously used arithmetic texts. With a vocabulary of 1327 words occurring in just one book of the five, there could be many words well taught by the teacher to her class that would be unknown to a mid-year entrant who has been consistently learning another third-grade arithmetic book vocabulary.

Teachers on textbook committees need to be aware of the differences in the size of vocabularies, the average new words per page, and the size of the quantitative vocabulary load on single pages, along with other features that make up a good textbook.

The results of this study point up the fact that much remains to be done on the problem of making elementary school arithmetic textbooks "readable" to children. Only one phase of reading difficulty (quantitative aspects of vocabulary load) has been explored in this research, which merely hints at the complexity of the total problem of satisfactory reading level of arithmetic texts.

EDITOR'S NOTE. Authors of arithmetic textbooks consciously try to keep the general reading difficulty of their books below the normal level of a reading textbook for the same grade. There are many elements in "level of reading difficulty" and the factor that is a stumbling block for one child may not be such for another. The five books examined included 3329 different words and yet a reading vocabulary of 4000 words is reasonable at the end of grade three but this reasonable number is appropriate for the average child and we do have these below-average people in all classes. A "new word" in an arithmetic text may not be a new word to the child. Certainly there are degrees of "newness" or familarity with words. It may be that an "old word" in one context approaches a "new word" in another context. The matter of analyzing reading is not simple. Perhaps we would do well to consider this more of a "gestalt."

The author points out how important it is for teachers to be conscious of the "new words" and provide for learning them. Certainly some of the hardship can be avoided if textbook writers are more careful in the number of "new words" introduced on one page. However, the purpose of an arithmetic textbook is to provide for the learning of arithmetic and language must serve this purpose. It is not too difficult to introduce new and technical words but the learning of these should not be assumed for all children. That is a job for a teacher.

The Division of Common Fractions

THEODORE S. KOLESNIK
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THERE HAS BEEN A GOOD DEAL of discussion recently in regard to the best method of teaching the division of common fractions. Some teachers believe that the common denominator method is the one that should be taught. Others feel that the inversion method is best if it is preceded or followed by an explanation of why the inversion is mathematically sound. Another group feels that the inversion method should simply be "algorized" with any rationalization being delayed until such time that the pupil is mature enough to understand the complex fraction that is involved in the rationalization. Still another group favors the teaching of all methods with the pupil selecting the one that he particularly favors.

This writer feels, however, that there are in reality only two basic methods, the *common denominator method* and the *complex fraction method*, and that inversion is but a shorthand version of the complex fraction method. I also believe that both methods do have a proper time and place for introduction in the arithmetic program.

The common denominator method is based on finding the lowest common denominator, and then actually employing division in arriving at the quotient. These are familiar procedures to the pupil as the L.C.D. has already been introduced to the pupil in the addition and subtraction of unlike fractions, and the division involved in the numerators has been taught in the process of division of whole numbers. I have found that the common denominator method has proven highly successful when the operations have been presented in the following progressive steps;

1. "Three-fourths divided by one-half."
2. "3 fourths divided by 1 half."
3. As fourths and halves are unlike, a lowest common denominator must be found, and then, "3 fourths divided by 2 fourths."
4. As the denominators are now alike and are only a designation of like quantities, or that any fraction with a denominator of 1 has the value of the numerator, it follows then, that, "3 divided by 2 is $\frac{3}{2}$ or $1\frac{1}{2}$."
5. "$\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\div\frac{2}{4}=\frac{3\div 2}{1}=3\div 2=\frac{3}{2}=1\frac{1}{2}$."

Steps one through four are the rationalization of the method, and are accompanied by the necessary explanations as they are written on the blackboard. Step five is the short, numerical algorism that is explained and employed after the meaning of the process has been established.

The validity of the complex fraction method rests on the fact that a division problem can be expressed as a fraction, and, also, on the fact that both terms of any fraction can either be multiplied or divided by the same number without changing its value. This last process should not be new to the pupil if this technique has been adequately explained when teaching the expression of fractions in their lowest terms, and in finding the L.C.D. when adding or subtracting unlike fractions. However, we teachers tend to forget that a complex fraction is really complex, and that to the eyes and mind of a fifth or sixth grader a complex fraction is a radical departure from the ordinary. It is also a process that requires at least some preparatory groundwork before it can be introduced and clearly understood by the average fifth grader. The method can be successfully presented, however, if this preparatory work is done and the following general procedure is pursued;

1. "$\frac{3}{4}\div\frac{1}{2}$"
2. As a division problem can be validly expressed as a fraction, it follows then, that,

$$\text{"}\frac{\frac{3}{4}}{\frac{1}{2}}\text{"}$$

3. **We know from past experience that both terms of a fraction may be either multiplied or divided by the same number without changing its value. However, before continuing two more additional facts must be supplied. First, that the denominator of the complex fraction should be multiplied by some number which will give us a product of one, and, second, that this number is the reciprocal of the fraction which is the denominator of the complex fraction. Therefore;**

$$\text{“}\frac{\frac{3}{4}\times\frac{2}{1}}{\frac{1}{2}\times\frac{2}{1}}=\frac{\frac{3}{4}\times\frac{2}{1}}{1}=\frac{3}{4}\times\frac{2}{1}=1\tfrac{1}{2}\text{”}$$

It is the lengthy and detailed rationalization of this process that makes it unacceptable to me as the method to be used in the initial introduction of the division of fractions in the fifth or sixth grade. It should be readily apparent that the so called inversion method is not a method in itself, but is actually the last step of the complex fraction method.

For those who advocate the inversion method in which the algorism is presented without any rationalization at all, why not use the cross-multiplication method? This method is just as meaningless and, perhaps, a little easier to grasp than inversion alone. The cross-multiplication algorism is as follows;

1. $$\text{“}\frac{3}{4}\div\frac{1}{2}\text{”}$$

2. Cross-multiply the numerator of the first fraction and the denominator of the second fraction to get the numerator of the product, and then cross-multiply the denominator of the first fraction and the numerator of the second fraction to get the denominator of the product. Therefore;

$$\text{“}\frac{3}{4}\times\frac{1}{2}\times\frac{6}{4}=\frac{3}{2}=1\tfrac{1}{2}.\text{”}$$

The initial introduction of any new process should follow a transitional pattern wherein past facts, fundamentals and knowledge can be applied to the new situation, and should be one that is readily understood by the greatest number of pupils possible. In my estimation the only logical process that fullfils all these requirements in the teaching of division of fractions is the common denominator method. The advocates of the inversion method, whether it is accompanied by an explanation or not, are asking that the pupil should forget most of the fundamentals learned in their past experiences with common fractions and to embark on new and unfamiliar ground. Using the common denominator method is, on the other hand, a transitional step that embodies the knowledge taught in all the preceding work with fractions. Another advantage is the fact that this method actually does involve division, which happens to be the process being taught.

After the meaning and process of the common denominator method has been fully understood by the pupil, I believe that the complex fraction method can then be introduced. This method could be taught late in the sixth grade, or even postponed until the seventh grade. Moreover, it is my opinion that the inversion method be employed by the pupil only after the entire process has been mastered. At such time it should be pointed out that the inversion is the last step in the process and that it is merely a short-cut technique to be used in lieu of the entire method.

EDITOR'S NOTE. Yes, division of common fractions and mixed numbers has been widely discussed. The "Common denominator method" as frequently explained may be misleading. For example:

$$\frac{3}{4}\div\frac{1}{2}=\frac{3}{4}\div\frac{2}{4}=\frac{3\div 2}{1}=3\div 2=\frac{3}{2}\cdot$$

The second step appears as though the numerators are divided and that the denominators (4 and 4) are also divided to produce the 1 in the denominator. The explanation is often given that "like fractions may be divided by dividing their numerators. This is like dividing any other like quantities such as feet or dollars." So why bother to place a figure *one* in the denominator at any step? Instead of writing the division of $\frac{3}{4}\div\frac{1}{2}$ as a complex fraction, some teachers prefer to explain as

$$\frac{3}{4}\times\square\div\frac{1}{2}\times\square$$

with a "box value" such that the product in the divisor becomes *one* and then use the usual agreements such as "any value divided by one . . . " and "you always get a product of *one* in the divisor if you multiply the original divisor by its inverse (reciprocal)."

A Mathematics Assembly Program

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EVEN IN THIS AGE OF SPECIALIZATION, a teacher of mathematics of grades 6 through 9 may be called upon to favor his school with an "assembly program" to be cast with members of his official class. The authors of the following 'play' used their routine assembly program assignment as a means of showing the whole school some of the "fun" aspects of mathematics, as a means of introducing some amusing, though very thought-and-discussion-provoking 'tricks,' as one more 'stone' cast at the wall of traditional resistance to mathematics.

To heighten the anticipation of the day when the program was to be presented and to contribute to the feeling that the audience was expected to participate, a "ESTIMATE-THE-NUMBER-OF-BEANS-IN-THE-JAR" contest was conducted 3 days prior to the day of the program; each student in the school being invited to submit his own estimate on a separate ballot. Needless to say, the teacher was besieged by theories and discussions as to the method of making an accurate estimate.

The props for the show itself are fewer and simpler than the number usually found in most school plays. They are:

1 movable blackboard, chalk, eraser—OR an easel with several sheets of paper, crayons
table
2 four inch strips cut cross grain from large package of crepe paper
Scotch tape
Scissors
2 Shovels or other digging tools (Optional)
1 blindfold
1 paper pad
1 marking crayon
6 number cards as follows—on 20″×24″ oaktag

I	II	III
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63	2 3 6 7 10 11 14 15 18 19 22 23 26 27 30 31 34 35 38 39 42 43 46 47 50 51 54 55 58 59 62 63	4 5 6 7 12 13 14 15 20 21 22 23 28 29 30 31 36 37 38 39 44 45 46 47 52 53 54 55 60 61 62 63

IV	V	VI
8 9 10 11 12 13 14 15 24 25 26 27 28 29 30 31 40 41 42 43 44 45 46 47 56 57 58 59 60 61 62 63	16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63	32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63

Act I

ANNOUNCER: Teachers, guests, and fellow students. Today, our class takes great pleasure in bringing to you something new in assembly programs. We may call it a mathematics assembly or perhaps "Math Magic," "Math Quiz Show," or "Math Hodge-Podge." Perhaps we should be very mathematical and call it our X-Assembly. Whatever the name, we hope it

will entertain you and possibly intrigue you. (Curtain opens.) Our first act features Professor Excellente, who through the aid of special powers and his carefully trained assistants, can guess any number from 1 to 63. (Professor takes a bow). To start our program, who from the audience would like to come on the stage and have the great Professor guess your number? (Announcer selects one person.)

PROF. EXCELLENTE: Now my good young man/lady, would you kindly choose any number. Please tell this number to my scribe so that she may show it to the audience. (Action.) Now as I point to each of the cards, kindly state by saying 'Yes' or 'No' whether or not your number appears on each card.

PROF. EXCELLENTE: Thank you. You may collect your prize on the way down.

ANNOUNCER: May we have another volunteer?

PROF. EXCELLENTE: (Repeat dialogue as directly above.)

ANNOUNCER: Now Professor Excellente would like to try an experiment. He would be greatly honored to be given an opportunity to guess the ages of some of our teachers! Do we have any volunteers among our distinguished faculty? (Take two more—guess ages.)
Thank you, Thank you.

(CURTAIN)

Act II

ANNOUNCER: Act two of our program is a short skit entitled, "Paul Bunyan, Folk Hero and Math Wizard." Our storyteller will proceed with the narrative. (Curtain opens.) (Girl reader is at front, right side of stage. Boy is on left side of stage, dressed in dungarees, shovel, etc.)

GIRL READER: Many years ago, Paul Bunyan owned one of the largest and richest gold mines in the Rocky Mountains. Because Paul was America's strongest and wisest miner, he was able to work this valuable gold mine all by himself. As Paul mined this gold vein, the excavation became deeper and deeper so that Paul felt the need for a conveyer belt to help him transport the raw ore from the deep pit up to the surface. He devised a belt to solve his problem. This conveyor belt was a mile long and certainly was bound to get a lot of wear. Paul, a good engineer, decided to make one turn in the conveyor belt so as to distribute the wear evenly on both sides. Here is a belt which resembles that conveyor belt as used by Paul Bunyan in his gold mine. (Show Moebius Strip.)

Many years passed and Paul's gold mine became deeper and deeper. By this time Paul decided to get help in this work from Happy Sam.

Soon it became obvious that the conveyor belt was not long enough for Paul's rich underground gold vein. Paul and Sam had a conference. Sam thought it would be a good idea if Paul added more length to the existing conveyor belt, but Paul disagreed. Paul decided to cut the conveyor belt lengthwise down the center. Sam was worried. He wondered what good it would do to have two halves of a conveyor belt.

So the next day Paul went down to the mine with his trusty hunting knife and began cutting. What do you suppose happened? (Pantomime on stage during reading.) Upon completion of cutting . . .

(CURTAIN.)

ANNOUNCER: The conveyor belt you just saw on stage is called a Moebius Strip. Try this mathematical trick at home. (Noise offstage.) Here comes Happy Sam now. I wonder what he wants. (Enter Happy Sam.)

HAPPY SAM: That Paul Bunyan. If he can double the length of his conveyor belt so can I. Only I'll be smarter. Instead of twisting the belt once, I'll twist it twice. Just like that. Now let's see what happens when I cut it down the middle. I'll cut it slowly . . . slowly . . . slowly . . . and now the final cut. What in the . . . Why that blankety-blank Paul Bunyan . . . . Just wait till I see him! . . . (Goes storming off.)

(CURTAIN)

Act III

ANNOUNCER: Professor Excaliber and his assistant, Excelsior, will now perform a very happy and lucky number trick. Introducing the boy wonder . . . Professor Excaliber! (Curtain goes up—board and chalk.)

PROF. EXCALIBER: Today, I shall ask for a member of the audience to volunteer to help perform a most interesting experiment. Young man, would you kindly tell me your lucky number from 1 through 9.

PROF. EXCALIBER: The great Professor Excaliber will give you a very great gift. Since you are so fond of this number . . . if you will go to the blackboard on my right. Please write down the following numbers . . . 1, 2, 3, 4, 5, 6, 7, 9 . . . now multiply this series of numbers by the product of your lucky number times 9. So then, if your lucky number is 4 . . . we will multiply 12345679 by 36.

ANNOUNCER: And now one more volunteer from the audience please . . . (Choose one.)

PROF. EXCALIBER: Young lady, I will guess an answer to a complicated series of arithmetic operations while my assistant writes them on the blackboard. To make sure I do not see what he is writing, I shall have him blindfold me. (Blindfold.) Ready now.

Take any number. Add ten. Double your sum. Subtract 18. Divide by two. Subtract the number you first thought of . . . and your answer is 1.

(CURTAIN)

Act IV

ANNOUNCER: Act four will consist of another short play about a fascinating character known as Slow Joe, the mental caterpillar, and his encounter with Professor Exasperated. Our storyteller will continue with the narrative.

NARRATOR: Slow Joe was a most peculiar fellow. He left school when he was only nine, and since that time he has developed an everlasting dislike for education. He learned how to add numbers, but never discovered how fractions were handled. He even boasts that he can multiply or divide any number at all by 2 but not by any other number. Slow Joe, poor fellow, hates even numbers on the left.

Recently a rich uncle . . . that is, Slow Joe's rich uncle, fearing for the lad's future, hired a learned professor . . . Professor Exasperated, to tutor Slow Joe in mathematics. At his first lesson with the professor . . . here is what happened . . .

(CURTAIN.)

PROF. EXASPERATED: Now Choe, ve vill see if you can multiply. Pliss, mit der piece of chalk, dis problem doink. 29×24. (Joe works problem while narrator speaks.)

NARRATOR: Now what do you think Slow Joe will do. Here is what he actually did. Joe first divided 29 by 2. Remember he does not like fractions so he left the fraction off. Then he divided 14 by 2 getting 7. Then 7 by 2, getting 3, then 3 by 2 getting 1. Tired of dividing by 2, he began doubling the 24, getting 48. Doubling 48 he got 96, doubling 96, getting 192, doubling 192, getting 348.

Slow Joe did not like even numbers on the left so he crossed out the lines containing even numbers on the left. He then added up his numbers on the right . . . getting 696. Proudly, he stepped back from his work and announced to the professor: "I have finished, the answer is 696."

PROF. EXASPERATED: I do not believe dot. (Works out multiplication—claps hand hand to head, faints.) Vot have I done to deserve dis!

(CURTAIN)

ANNOUNCER: The strange thing about this method of multiplication is that all multiplying can be done just the way Slow Joe did it. Try it sometime. Now we are going to announce the grand prize winner of our GUESS-THE-NUMBER-OF-BEANS Contest.

(Two program girls enter and announce all contest winners.)

A Combined Content-Methods Course for Elementary Mathematics Teachers

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COLLEGES AND UNIVERSITIES have the responsibility of training prospective elementary teachers in the teaching of elementary mathematics. These teachers must be trained so that they can teach elementary school mathematics for meaning and understanding. In addition to training prospective elementary school teachers, the colleges and universities also have the responsibility of acquainting teachers in-service with the "newer" approach in teaching elementary mathematics. In this article, I will use "elementary teachers" as including both groups.

What does the elementary teacher have to know in order to teach elementary mathematics for meaning and understanding? For this discussion, I am going to consider three categories—fundamentals of elementary mathematics, social use of elementary mathematics, and psychological principles in teaching elementary mathematics.

The knowledge of the fundamentals of elementary mathematics includes more than just operational skill in computation. Elementary teachers must know the principles, generalizations, and relationships that make up elementary mathematics. They must thoroughly understand elementary mathematics as a unified logical system of ideas. In general education, elementary teachers are also concerned with the social use of elementary mathematics. This means that they must know how elementary mathematics is used in the child's everyday life as well as in his future needs. In the teaching of elementary mathematics, elementary teachers must know how to apply the principles presented in Psychology and its related studies. This involves a knowledge of the laws of learning, principles of Child Development, principles of social psychology, etc.

The next question is how to present the necessary knowledge to elementary teachers. In this article, I wish to consider the aspects of training elementary teachers in the fundamentals of elementary mathematics and the principles and methods (or techniques) of teaching elementary mathematics. There are two prevalent ways of carrying out this training. One of these is to have separate courses—Fundamentals of Elementary Mathematics, usually taught by a Mathematics staff member, and Methods of Teaching Elementary Mathematics, usually taught by an Education staff member. Usually the Mathematics course is a prerequisite for the Education course. The other way is to have a course which combines both content and methods, taught by a staff member who has knowledge of both fields.

Let me illustrate the development of a topic in a combined content-methods course. For the illustration, I'll take the topic of column addition (three-one digit addends, sum less than 10) which is usually presented in the second grade. In the combined course, the discussion would bring out the background topics essential to the development of column addition. A suitable problem situation (such as one involving 2 marbles+3 marbles+1 marble) would be discussed, as well as the solution of it on various levels—concrete, picture, semi-concrete, and abstract. During the discussion it may help to have some of the elementary teachers show and state (if possible on second grade language level) the different groupings. Elementary teachers must realize that the statements made by second graders in reference to the different groupings would be at first lengthy and involved. For example, they might say "we first put together the two marbles and the three marbles which are

five marbles and then we put together the five marbles and the one marble and we have six marbles altogether." From this long oral statement and similar ones giving the other ways of grouping, the possible intermediate written statements, before $(2+3)+1$ and $2+(3+1)$ are reached, would be brought out. Various drawings of marbles and tally marks could show the different groupings. Boxes may help to indicate the groupings discussed orally.

$$00+000+0$$

$$\boxed{00+000}+0 \quad \text{or} \quad 00+\boxed{000+0}$$

$$00000 \quad +0 \qquad\qquad 00+0000$$

$$000000 \qquad\qquad\qquad 000000$$

In this development, there would be a discussion on the shortening of notation and terminology. This would involve the gradation of 2 marbles to the abstract notation of 2 (using 2 ones as an intermediate step on the semi-concrete level where tally marks are used to represent the marbles). The terminology would gradually be shortened to "parentheses, two plus three, end of parentheses, plus one," written with symbols as $(2+3)+1$. Transfer from the horizontal to the vertical algorism would be made with the discussion of unseen numbers, rearranging, and regrouping of numbers. The elementary teachers could "see" why, in column addition, addition should be carried out both down and up. Showing the solutions of $(2+3)+1$ and $2+(3+1)$ on the number line could be the next topic of discussion. The statement $(a+b)+c=a+(b+c)$ where a, b, and c are any positive integers or zero, could be "generalized" (not in the sense that it necessarily follows from the examples but rather as inductive reasoning).

The above brief description should point out that the combined course presents an integrated picture of the three categories discussed beforehand. This is essential since in the actual teaching, a skillful teacher will have to fuse together the "why, what, and how." The elementary teacher will also see the vertical development of each concept. Thus the teacher can see the initial stage of development (readiness) and also various stages of subsequent development. In the above illustration (column addition), the teacher can see the initial stage of development possibly in the first grade and the succeeding development in the second through the sixth and seventh grades.

This method of organization also satisfies the immediate need and interest of the elementary teacher. At the time they are taking this course (or courses), these students are either student teaching, teaching in the field, or anticipating student teaching in the near future. Thus the question, how would you develop the concept of column addition in the elementary grades, is a pertinent one. In taking the procedure as described above, they are learning the answer to the question as well as learning the fundamentals of mathematics involved in that concept. It is a painless way to teach them the fundamentals of elementary mathematics. The reason why it is painless is that the mathematical ideas are developed from the concrete to the abstract. Quite a number of the students in these courses have to "see" the principles of commutation and association. The manipulation of the groups of marbles in the "guise" of how this can be shown clearly to first and second graders is necessary for some of the college students to understand the principles themselves. In short, they *learn* it. This is essential since tests show that the students are extremely low in elementary mathematics. After the development of the concept of column addition, procedures for drill and application are discussed. For students who are low in these aspects, practice exercises and problems are assigned. Thus the principle of learning, "drill and application should follow after the meaning and understanding of a concept is reached," is carried out even on the college level.

The author has found that in teaching separate courses, the elementary teachers usually do not get a deep understanding of the fundamentals of mathematics and the principles of learning. In the former, quite often, they are given practice exercises such

as $30+42+342=?$ 3% of $30=?$, and $3\frac{1}{4} \div 2 =?$. In the latter, the principles of learning are often given without specific suggestions in the teaching of certain concepts in elementary mathematics. In the treatment of specific topics, quite often the separate courses give isolated bits of information, which they memorize. For example, "regrouping the addends does not change the sum. This is known as the principle of association with respect to addition. In symbols, this is $(a+b)+c=a+(b+c)$." "The teaching of column addition of three addends follows the teaching of two addends." How will the above approach achieve a deep *understanding* (I mean understanding) of the fundamentals of elementary mathematics and help the teacher to *develop* it in the classroom?

It thus seems to the writer that the four statements often given as reasons or goals for the "new" approach in teaching elementary

1. Efficiency in learning.
   The combined content-methods approach brings about a greater amount, depth, and integration of knowledge in in the amount of time we can allocate in teacher training of elementary mathematics.
2. Retention of learning.
   The greater depth and integration of knowledge achieved by the combined content-methods course results in better retention. This is important since the prospective teacher may teach next year or three years later.
3. Application (or transfer) of learning. The combined content-methods course with its emphasis on the integration of the three categories will result in better teaching in the actual classroom. This outcome is due to the fact that in the actual classroom there is a fusion of the three aspects (mathematical, social, and psychological.)
4. Attitude of the learner.
   A combined content-methods course will bring about a better attitude toward the teaching of elementary mathematics. One of the reasons that elementary teachers are afraid to teach elementary mathematics is that they don't understand it themselves. The combined course with its emphasis on presenting a vertical development from the concrete to the abstract (which at times includes algebraic representations, generalizations, principles, and relationships) will result in understanding elementary mathematics. Understanding aids in confidence, interest, and attitude. Confidence, interest and attitude of the elementary teacher is contagious. We want the children to like mathematics and to have an interest in continuing in mathematics as they go to high school and college.

By this time, the reader may question as to the "proof" for the arguments given in behalf of a combined content-methods course. I can only offer my experience with both methods of approach and the numerous discussions with prospective elementary teachers and teachers in the field. It is hoped that this article will foster "thinking" on the part of teachers of these courses, supervisors and directors of elementary education. In that way we can better analyze the pros and cons and possibly set up experiments to test our hypotheses. Our goal should be to improve the training of elementary teachers.

EDITOR'S NOTE. The author argues for and has demonstrated through his own experience that it is better for prospective teachers to combine the content and method (educational procedures) of arithmetic into a single course. He is dealing with the various elements of insight and knowledge that a teacher of arithmetic should possess before facing a group of children. He is not here considering the other realm of mathematics which she should have as a part of her General Education. Twenty-five years ago some normal schools were using "professionalized subject matter" which was very similar to the author's idea of the needs of teachers.

In the February, 1932 issue of *The Mathematics Teacher* the editor made the following summary of three types of courses then current.

(Concluded on page 158)

How Well Do 158 Prospective Elementary Teachers Know Arithmetic?

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FOR THE PAST SEVERAL YEARS the College of Education of Southern Illinois University has required its students majoring in elementary education to take a course known as Mathematics 210, which is described in the *University Bulletin* as a "professional treatment of the subject matter of arithmetic methods and a study of trends and current literature on the teaching of arithmetic." This course is offered by the Mathematics Department of the College of Liberal Arts and Sciences and carries four quarter hours of credit. Its prerequisite is a general mathematics course which does not count toward a major or minor in mathematics but which does include, however, a careful study of the real number system and other topics providing a better understanding of arithmetic and elementary algebra.

Deficiencies in Subject Matter

Inasmuch as Mathematics 210 was being recently requested by more students than could be accommodated by the teacher to whom the course had been regularly assigned, the writer was given the responsibility for teaching one section of this course each quarter, the first in the summer of 1955. It had been the experience of the regular teacher that much of the class time which should have been used for the teaching of methods had to be consumed in an effort to overcome deficiencies in arithmetical principles and processes which the students should have presumably mastered before entering the methods course. In an attempt to discover the extent of these deficiencies and to obtain information which would allow instruction to be adapted to the needs of the students, the writer prepared and arranged to have given to the Mathematics 210 students a test consisting of 40 items which he considered representative of the knowledge prospective teachers of arithmetic should possess.

In all cases the test was administered at the first class meeting of each section. Just before the students began to write, they were told that performance on the test would be used in an attempt to adjust instruction to their needs and would not be counted in arriving at the term grade in the course. They were encouraged to answer the items in any order that appealed to them and to do their very best such that responses would reflect a rather accurate picture of the knowledge possessed on this particular test. The test provided room for recording only the answers and was scored by giving one point for each item answered correctly.

At the second class meeting the student, filled out an information form on which he gave, among other things, his age, high school from which graduated, date of graduation, mathematics courses studied in high school with number of semesters of credit earned in each, colleges attended, college classification, mathematics courses studied in college with the number of quarter hours in each, and years of teaching experience showing subjects taught or grade levels at which the teaching was done. It will be the purpose of this article to deal with the data obtained from the 158 students who took the test and filled out the information forms.

Test and Performance

Table 1 lists the items in the order in which they appeared on the test and gives the number and per cent of students answering each item correctly. This table also ranks the items according to difficulty of performance by assigning a rank of one to that item missed by the greatest number of students.

TABLE 1

CORRECT RESPONSES OF 158 COLLEGE STUDENTS ON AN ARITHMETIC TEST OF 40 ITEMS

Item Number	Items Listed in Order of Appearance on Test	No. of Students Giving Correct Answers	Per cent of Stud. Giving Correct Answers	Item* Rnk. in Ord. of Difficulty
1	Write in figures: Six billion sixty million six hundred thousand sixty-one	106	67.1%	30.5
2	Write in figures: Five thousand and six hundred twenty-five thousandths.	72	45.6	19
3	$12\frac{1}{2}+\frac{7}{8}+4\frac{2}{3}$	114	72.2	35
4	$2.35+\frac{7}{8}+52\frac{3}{4}$	87	55.1	25
5	$15\frac{1}{4}-11\frac{1}{3}$	142	89.9	40
6	$7\frac{1}{2}+9\frac{1}{3}-12\frac{7}{8}$	94	59.5	27
7	$256.75-212\frac{5}{6}$	71	44.9	18
8	$(24)(36\frac{3}{4})$	116	73.4	36
9	$8\frac{1}{3}\div 18\frac{3}{4}$	106	67.1	30.5
10	$\frac{11}{12}\div 55$	111	70.3	33.5
11	$(48\frac{1}{4})(56\frac{5}{8})$	60	38.0	14
12	$(\frac{5}{8})(\frac{4}{15})(0.33\frac{1}{3})$	55	34.8	12
13	$1728\div 0.144$	96	60.8	28
14	$0.1728\div 144$	118	74.7	37
15	$\frac{5}{12}/\frac{3}{8}$	108	68.3	32
16	(25)(5)(0)(3)	82	51.9	24
17	(7)(2)(0)+(3)(5)−(2)(0)(2)	78	49.3	23
18	(17.5)(0.202)	123	77.8	38.5
19	729−0.2456	123	77.8	38.5
20	$\frac{3}{8}+\frac{1}{4}/\frac{2}{3}-\frac{1}{6}$	89	56.3	26
21	What part of $\frac{11}{12}$ is $\frac{5}{8}$?	29	18.3	3
22	Find $\frac{3}{8}$% of 240.	33	20.9	5
23	25% of 72 is what per cent of 15?	39	24.7	6

TABLE 1 (*Continued*)

Item Number	Items Listed in Order of Appearance on Test	No. of Students Giving Correct Answers	Per cent of Stud. Giving Correct Answers	Item* Rnk. in Ord. of Difficulty
24	If 21% of a number is 84, find 25% of the number.	63	39.9	16
25	Find the simple interest on \$250.50 for one year and six months at $4\frac{1}{2}$%	49	31.0	9
26	A suit which costs \$36.00 is sold to gain 25% on the cost. Find the selling price of the suit.	111	70.3	33.5
27	How many feet of fencing will be required to enclose a circular flower bed 8 feet in diameter?	52	32.9	11
28	A person drives a car 350 miles at an average speed of 50 miles per hour, and uses 20 gallons of gasoline on the trip. Find the average mileage per gallon.	105	66.5	29
29	A room 21 feet long and 15 feet wide is covered with carpet costing \$12.50 per square yard. Find the cost of the carpet.	51	32.3	10
30	A truck loaded with coal weighs 12,550 pounds. If the empty truck weighs 5100 pounds, find the cost of the coal at \$8.60 per ton.	47	29.7	7
31	In a certain division exercise, the quotient is 18, the divisor 46, and the remainder 12. Find the dividend.	77	48.7	22
32	How much will a housewife pay for $7\frac{1}{2}$ lineal feet of goods costing \$2.60 per yard?	66	41.8	17
33	Find the cost of 580 gallons of fuel oil at 12.8¢ per gallon.	75	47.5	20.5
34	The assessed valuation of a person's real property is \$3650.00. If the tax levied against his property is \$4.56 per \$100 of assessed valuation, how much tax will he pay?	62	39.2	15
35	If the minuend is 524 and the difference is 216, what is the subtrahend?	56	35.3	13
36	Paint can be bought in gallon containers at \$5.60 per gallon. If bought in pint containers, it will cost \$0.75 per pint. How much does one save on the gallon by buying in the large containers?	75	47.5	20.5
37	Ready-mixed concrete costs \$14.50 per cubic yard. Find the cost of the concrete required to make a drive 120 feet long, 9 feet wide, and 6 inches thick.	30	19.0	4
38	Write $0.56\frac{1}{4}$ as a common fraction in lowest terms.	25	15.8	1
39	Find the perimeter of a trapezoid whose parallel sides are 14 and 20 feet respectively and whose non-parallel sides are 8 feet and 10 feet.	48	30.4	8
40	The product is $106\frac{5}{8}$ and the multiplier is $8\frac{1}{2}$. Find the multiplicand.	26	16.5	2

* The item missed by the greatest number of students was considered the most difficult and was assigned a rank of one.

Below are some of the observations which an examination of the Table seems to justify:

1. Item No. 5, which involves the subtraction of mixed numbers without borrowing, is the least difficult. Nevertheless, 16 students, or 10 per cent of all who took the test, failed to answer this item correctly.
2. Item No. 38, which requires the changing of a rather simple decimal to a common fraction, proved to be the most difficult. Actually, 133 students, or about 84 per cent of all, missed this item.
3. Performance on verbal problems, or thought problems as the writer prefers to call them, was much poorer than than that on the other items. Of the 20 items which may be classified as thought problems, 15 had rank numbers smaller than 20, whereas only 5 had rank numbers larger than 20.
4. Items involving percentage concepts present considerable difficulty. Of the 6 items in this category, 4 had rank numbers smaller than 10, another smaller than 20, and only one larger than 20.
5. The performance on items pertaining directly to life situations was rather poor. In no instance did as many as half the students who took the test answer correctly any one of those items dealing with situations fairly closely related to the area from which these students come.

Performance with Respect to Teaching Experience and College Classification

Table 2 gives the performance of the 158 students distributed according to teaching experience and classification in college. The following statements are based upon data presented in this table:

TABLE 2

NUMBER OF TEST ITEMS ANSWERED CORRECTLY BY 158 COLLEGE STUDENTS DISTRIBUTED ACCORDING TO TEACHING EXPERIENCE AND CLASSIFICATION IN COLLEGE

Test Items	All Students	Distribution with Respect to						
		Teaching Experience		College Classification				
		None	One Yr. or More	Freshman	Sophomore	Junior	Senior	Other*
35–39	10	2	8	0	2	2	6	0
30–34	19	10	9	0	6	8	4	1
25–29	20	10	10	1	8	7	4	0
20–24	23	18	5	1	15	4	3	0
15–19	30	16	4	1	11	15	2	1
10–14	35	32	3	1	21	7	5	1
5–9	17	12	5	1	4	7	5	0
0–4	4	4	0	0	2	1	1	0
Total No. of Students	158	114	44	5	69	51	30	3
Mean No. of Items Answered Correctly	20.0	17.6	25.6	17.0	19.1	20.1	22.4	21.3

* One of these students was a postgraduate and the other two were unclassified.

TABLE 3

NUMBER OF TEST ITEMS ANSWERED CORRECTLY BY 158 COLLEGE STUDENTS DISTRIBUTED ACCORDING TO HIGH SCHOOL AND COLLEGE PREPARATION IN MATHEMATICS

Test Items	Mathematics Preparation in High School and College																			
	1 Year of H. S. Plus Quarter Hr. Coll.					2 Years of H. S. Plus Quarter Hr. Coll.					3 Years of H. S. Plus Quarter Hr. Coll.					4 Years of H. S. Plus Quarter Hr. Coll.				
	0	4	8	12	16	0	4	8	12	16	0	4	8	12	16	0	4	8	12	16
35–39		1				1	1	1				2			1		1	2		
30–34							3	6				3	3		2		1	1	1	
25–29		1				2	5	3				3		1	1		4			
20–24		5					5	1	1			4	4	1	1		1			
15–19	1	2					18	1				4	3							1
10–14		10				1	16				1	6	1							
5–9		4					9	3			1									
0–4		3					1													
Total No. of Students	1	26				4	58	15	1		2	22	10	2	5		7	3	1	1
	27					78					41					12				
Mean No. of Items Answered Correctly	17.0	14.6				25.4	16.6	25.2	24.0		10.0	22.2	20.9	29.5	30.0		28.7	35.3	31.0	19.0
	14.6					18.9					22.8					29.9				

1. The mean number of items answered correctly is 20.0, or just one-half the total number in the test. Although it is not revealed in the table, nevertheless, the study shows that one student answered just one item correctly, and only one answered as many as 39.
2. Students with teaching experience did significantly better than those without. The mean number of items answered correctly by the experienced group[1] was 25.6 as compared with a mean number of 17.9 for the inexperienced.
3. Performance becomes progressively better as the level of classification in college becomes higher. The 5 freshmen answered correctly a mean number of 17.0 items; the 69 sophomores, 19.1; the 51 juniors, 20.1; and the 30 seniors, 22.4.
4. Perhaps it is safe to assume that better performance in the higher classifications can be attributed to both teaching experience and college training. However, the overlap resulting from the fact that 40 of the 81 juniors and seniors who took the test were experienced teachers[2] makes it difficult to determine just how much of the improvement is caused by each of these factors.

Performance with Respect to Preparation in High School and College Mathematics

Table 3 shows the performance of the 158 students distributed according to their qualifications in high school and college mathematics.[3] Listed below are some points based on a study of this table:

[1] Of the 44 students with teaching experience, 33 had taught from 1 to 9 years, 7 from 10 to 19 years, and 4 for 20 or more years.

[2] The experienced teachers are classified as follows: freshmen 1, sophomores 3, juniors 16, seniors 24.

[3] If the student had only one year of high school mathematics, the course was elementary algebra in about three-fourths of the cases and general mathematics in the rest. If the student had only two years of high school mathematics, in three-fourths of the cases the courses were one year of algebra and one year of plane geometry, and the rest a combination of algebra and general mathematics. For those with three and four years, the additional courses were advanced algebra, trigonometry, solid geometry, and business arithmetic. If the students had only four hours of college credit, the course was the prerequisite described at the beginning of this article. Additional college credit represented courses such as trigonometry, college algebra, analytical geometry, and calculus.

1. Performance becomes better as preparation in mathematics increases. The 27 students having only one year of high school mathematics and four quarter hours of college mathematics did the most poorly on the test, having answered correctly a mean number of 14.6 items. A comparable figure for the 78 students with two years of high school mathematics is 18.9; for the 41 with three years, 22.8; and for the 12 with 4 years, 29.9.
2. Those students with more than two years of high school mathematics performed significantly better than did those with two years or less. Just how much of the improvement can be attributed to the additional high school preparation and how much to other factors cannot be satisfactorily determined. Since very few, if any of the high schools from which these students come, require more than two years of mathematics for graduation, it is very likely that students with credit beyond the two years, took the courses as electives. This would indicate a special interest in, or a direct need for the subject, either or both of which would motivate the student to do good work therein and hence to perform better on this test.
3. Students having more than four quarter hours of college credit in mathematics performed better than did those having just the prerequisite hours. Of the 38 students in the former category, 29, or 76 per cent of the group, made scores above the mean, whereas, only 9, or 24 per cent made scores below. Perhaps it should be pointed out in this connection that 16 of these 38 students had teaching experience, which may have contributed to better performance.

How Well Do These 158 Students Know Arithmetic?

A careful consideration of the data presented in this article inevitably leads one to answer the question posed as its title by stating that far too many of the 158 prospective elementary teachers studied herein have an insufficient knowledge of arithmetic to teach the subject effectively. If the deficiencies were just confined to this particular group, the ultimate results will not be so damaging in general, although extremely serious from the standpoint of the effects upon the boys and girls who will eventually come under the influence of these teachers. According to the writer's experience and investigations, the alarming thing from the standpoint of the future of mathematics is the likely existence of many comparable deficiencies in other parts of the nation.

Good teachers develop in their pupils an enthusiasm for arithmetic, but poor teachers often cause their pupils to have an apathetic attitude toward, or even a pronounced dislike for this important subject. In this age of science and technology, an adequate knowledge of arithmetic is an essential part of general education, and the specialists in mathematics are becoming more and more necessary. Poorly prepared teachers are not likely to provide the stimulus which will inspire their pupils to acquire this knowledge and arouse in them the desires to pursue other branches of mathematics. If the objectives of education in our democracy are to be more fully realized, it seems rather obvious that something must be done to produce more good arithmetic teachers. Fortunately, some of those responsible for the training of elementary teachers have taken steps toward this end. If the others will follow this example, the writer is of the opinion that a significant and widespread improvement in the understanding and application of arithmetical principles and processes will soon be in evidence throughout the country.

(*Editors Note on page 158*)

Arithmetic Books for Elementary Schools

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The purpose of this paper is to summarize an analysis of ten different series of arithmetic textbooks together with their accompanying workbooks and teacher's guide books. General patterns and new developments are described.

Most of the textbooks follow a chapter plan or organization that is not necessarily a unit plan of organization. Topics are generally divided into eight to twelve chapters covering approximately 250 to 350 pages in all. Most of the authors of the textbooks are recognized leaders in the field of elementary school arithmetic.

Content in Texts

The content of the books varies somewhat from grade to grade. Distribution of topics shows little change in the past five years. There is certainly more agreement than disagreement as to practice in grade placement. Perhaps some of the more common agreements may be cited in general terms in order to give an overall view of the elementary arithmetic program in modern textbooks:

Readiness: Recognizing and writing numbers 1–10.

First Grade: Addition and subtraction facts, 1–10; symbols for the fundamental processes; some quantitative ideas and terms; simple measurement (time, ruler, shapers, directions); counting by ones, twos, fives, and decades; reading and writing numbers to 100; small coins of money; fractions (one-half) and problem solving.

Grade Two: Counting to 1000; reading and writing numbers to 1000; place value to 100; addition and subtraction facts, 11–18 or 19; further money study; fractions (one-half; one-fourth); comparative terms and measurement; calendar time (days, weeks, months, years); problem solving.

Grade Three: Further arithmetic vocabulary development; reading and writing numbers to 10,000; place value to 1000; simple geometric recognition; further measurement (capacity, length, quantity, and time); all coin and paper moneys; addition and subtraction of money; unit and non-unit fractions with denominators of 2 through 8; Roman numerals through 10 or 12; basic facts of addition, with and without carrying, all groups (1, 2, 3 figure numbers); basic facts of subtraction, with and without borrowing; multiplication facts through products of 1 through 20 or 36 or 45; division facts through dividends of 20 to 36 (divisors 2, 3, 4, 1).

Grade Four: Reading and writing numbers through millions; place value; Roman numerals to 40 or 50; average; basic multiplication facts, products 40–81; multiplication of 1–2 figure numbers by 1–2 figure multiplier; basic division facts, 40–81; dividing 2–3 figure dividends by any digit; division by 1–2 figure divisor, with quotient 2 figures; measurement (temperature, liquid capacity, simple dry capacity; weight; gross); multiplication and division with money; fractions (improper mixed), easy steps in addition and subtraction of like fractions; problem solving with two-step problems.

Grade Five: More Roman numerals; diagnosis and reteaching and extension into 5 place numbers; multiplication by 2–3 figure multiplier of a 3–4 place number; division by 2-figure divisor with a 1-2-3 place quotient; 3-figure divisors, optional; zeros in divisor and dividend; graphs and scale drawing; addition and subtraction of proper fractions and mixed numbers, unlike; addition and subtraction of decimals, tenths and hundredths; perimeter and area; problem solving with three-step problems.

GRADE SIX: Diagnosis, reteaching, and extension into larger figures; rounding and estimating; 3-figure divisors; addition, subtraction, multiplication and division of common fractions; addition, subtraction, multiplication and division of decimal fractions; short division (optional); addition, subtraction, multiplication, and division of denominate numbers; personal accounts and budgets; percentage (optional).

Physical Features of Texts

There is generally one book for each grade and often two or more books for the first grade. New books are including a larger number of pages for the first grade program. There are a few "number readiness books" such as found in the reading program. Generally these aim toward building number concepts, teaching the child to recognize and be able to write the numbers 1 through 10, and to stimulate interest. The readiness materials and other first grade books often have accompanying parts such as materials for handling, arranging, and discovering. Sometimes the books for the first grade are divided into primer level and first grade level. Adequate space is generally provided where pupils are asked to write or draw in the book. Number reading charts, number cards, films, and filmstrip often serve prior to the use of a text. Books for grades one and two are generally paperbound. Books for the third grade through the sixth grade generally have hard covers which are of good quality and frequently are colorful. Grade placement is frequently indicated by a number or by a symbol. The pages are of good quality material. The type is large and clear. Pictures are well placed and often in color.

The illustrations are intended as learning aids. They show basic data and the action involved in a solution. Pictures are accurate presentations, with much attention given to details to avoid wrong impressions. The art work shows evidence of collaboration between author, artist, and editor. Color is used as a helpful guide in directing attention and as a means of emphasizing important points and things to remember in many of the books.

Points of Emphasis by the Texts

A survey of the recent textbooks would lead one to note there is considerable emphasis placed on the following features:

a. A sincere effort is made to deepen the teacher's understanding of arithmetic.
b. Pupil understanding of the number system is held as a prime objective.
c. Pupil understanding of what he is doing is demanded from the very beginning stages.
d. Skill and accuracy in fundamental operations is stressed.
e. Visual method of presentation of work is widely used.
f. Rules are generally arrived at in a deductive manner.
g. There is little emphasis on the "speed test" type of material.
h. Immediate use of the skill to be mastered is provided through interesting settings in the initial presentation.
i. Specific helps are being offered pupils in learning facts and in recalling important information.
j. Various solutions are encouraged for checking understanding of arithmetic facts, processes, and generalizations.
k. Novel settings are arranged for reintroduction of ideas at more advanced levels.
l. Attempts are made to help pupils develop skill in analyzing problem situations.
m. The reading problems are based on social situations within experience of most children.
n. Suggested enrichment material for the "fast pupil" on various topics relating to the arithmetic program is often suggested. Such work is often labeled as "those with no work to correct" or "topics to look up" or "special assignment." Too, short cuts are often recommended for the above-average pupil. On the other hand, special drill material is provided for those who need more practice.
o. A textbook is recognized as only a part of the total program of instruction.
p. An enlarged review is given at each successive grade level. Reviews and summaries are recognized as vital instructional procedures.
q. Reading difficulty has been kept to a minimum. The vocabulary is carefully controlled and sentence form is simple. Use of headings and other such aids improves readiability.
r. The everyday uses of arithmetic are mentioned in association with the work. Estimation and rounding receive considerable attention along with opportunities of applying reasonable checks on computation. Non-pencil-and-paper situations are posed.
s. Suggestions are offered for capitalizing upon incidental phases of arithmetic.
t. Tables of measures remain a stable item in most arithmetic textbooks.
u. Measurement is frequent (informal check-ups) and systematic (for example, inventory tests at beginning of year, readiness test for big, new topics.

Workbooks

There is usually one workbook for each textbook which covers the same concepts for the level, but attempts to make a little different approach from the textbook. There has been a noticeable attempt toward avoiding making the workbook a duplicate of the textbook. The pages in the workbook are often keyed to pages of work in the textbook and often the teacher's edition gives information for use of the workbook as well as for the textbook. Frequently enrichment work is designed especially for the brighter pupil.

Manuals

Most of the "teacher guides" reproduce a facsimile of the pupil's page reproduced on the same page as the teacher's plans. These manuals generally include objectives for the year and for the units of work; background information for the teacher; and an overview of the program which gives the author's theory of teaching arithmetic or his point of view concerning materials by the chapter or unit (research is at times presented to support a particular philosophy). The new arithmetical vocabulary for the year, the meanings of which may be derived through pictures, questions, word lists, or definitions; answer key for exercises (or answers may be over printed on the pages reproduced of pupil's lesson page), and materials needed for the lessons are also included. Provision for individual differences vary from general comments, to teaching suggestions broken down into parts—for the class as a whole, for the able pupil, and for the slow learner.

Most manuals offer a helpful chart showing the organization of the arithmetic content of the textbook series. Sometimes the basis used for selection of the content for the series is explained.

Some manuals offer the author's estimate of the rate of progress to be expected during the year as a suggestive guide for the teacher. Recreational arithmetic, riddles, and puzzles, are often a part of the teacher's guide. Description of how to construct teaching aids may be found in some guides. Sources of all sorts of aids—charts, posters, cards, films, filmstrips, books—may be indicated. References may be given to distributors of material. Bibliographies of professional materials and readings are usually a part of the manual.

A few manuals suggest preparatory activities to use prior to the textbook. All sorts of things may be found in manuals; general suggestions for grading papers; using community resources in arithmetic teaching; and provisions for "notes for the teacher to write."

The manual, whether it comes in spiral or hardback or some other binding, is more than just a book of recipes—it usually attempts to explain the reasons for the suggestions offered. The manual is an unparalleled source book for the busy teacher.

To the reviewer, these are the things of note in the new arithmetic textbooks, workbooks, and teacher guide books. Class instruction will be influenced by these trends to the extent that teachers become aware of and accept the ideas contained within them.

EDITOR'S NOTE. The author has indicated in a general way the materials included in American textbooks in arithmetic. This information is in harmony with most standard courses of study. It is interesting to observe how important courses of study influence textbooks and how in turn they are influenced by the textbooks. Workbooks are much less common now than they were twenty-five years ago. Teachers Manuals have grown in popularity. These manuals can be of great service to a teacher, particularly if she is fairly new at her work or if she is one who is returning to teaching after a considerable interim. These manuals occasionally are good "advice books" because they not only indicate methods of conducting learning but they also explain the significance of subject matter to the teacher.

Divisibility by Odd Numbers

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TESTS FOR DIVISIBILITY by seven and thirteen were described by Francis J. Mueller in the November 1958 issue of THE ARITHMETIC TEACHER. In addition, proofs for these tests were presented in the above-mentioned article. However, the method of deriving such rules was omitted. When this method is known, the tests may be extended to include divisibility by any number ending in 1, 3, 7, or 9. Since divisibility by numbers ending in 5 may be ascertained on sight, a means is thus provided for determining exact divisibility by any odd number.

Method of Deriving the Tests

1. Find the lowest multiple, ending in 1, of the number under consideration.

For numbers ending in 1, use the number itself.
For numbers ending in 3, multiply by 7.
For numbers ending in 7, multiply by 3.
For numbers ending in 9, multiply by 9.

2. Use each multiple to formulate two rules, similar to those proposed by Francis J. Mueller for 7 and 13.

For 7 the multiple is 21, (7×3). The-ratio between the units-digit and the non-unit digits of this number is used to devise one addition and one subtraction rule. The subtraction rule was given in Professor Mueller's article so it will not be repeated here. The corresponding addition rule follows:

ADDITION TEST FOR DIVISIBILITY BY SEVEN: A whole number is exactly divisible by 7 if the sum obtained by adding five times the units-digit to the number formed by its non-unit digits is exactly divisible by 7.

The *five* in "five times the units-digit" is obtained by subtracting the 2 in 21 from 7.

Two Tests for Divisibility by Nineteen

The method of deriving the tests will be illustrated in the following example.

1. The required multiple of 19 is 171, (19×9).
2. The ratio of 17:1 found in the number 171 is used in the first or subtraction test.
3. The ratio of $(19-17)$:1, that is, 2:1 is used in the second or addition test.

SUBTRACTION TEST FOR DIVISIBILITY BY NINETEEN: A whole number is exactly divisible by 19 if the difference between seventeen times its units-digit and the number formed by its non-unit digits is exactly divisible by 19.

Example: Test 1653 for divisibility by 19.
Seventeen times the units-digit is 51, (17×3).
The number formed by the non-unit digits is 165.
The difference between 165 and 51 is 114, which is divisible by 19.
Therefore 1653 is exactly divisible by 19.

ADDITION TEST FOR DIVISIBILITY BY NINETEEN: A whole number is exactly divisible by 19 if the sum obtained by adding twice its units-digit to the number formed by its non-unit digits is exactly divisible by 19.

Example: Test 1653 for divisibility by 19.
Twice the units-digit is 6, (2×3).
The number formed by the non-unit digits is 165.
The sum of these is 171.
Repeat the test:
Twice the units-digit is 2.

The number formed by the non-unit digits is 17.

The sum 19 is, of course, exactly divisible by 19.

Thus 1653 is exactly divisible by 19.

In this case, the addition test is simpler and easier to use than the corresponding subtraction test. In fact, for numbers ending in 3 or 9 the addition test is to be preferred. For numbers ending in 1 or 7 the subtraction test is more useful. However, in testing some examples a combination of the two tests can be used to advantage.

Proofs of Divisibility

Professor Mueller's proofs are extended here to include divisibility by all odd numbers ending in 1, 3, 7, or 9. Only two proofs will be given since the others may be inferred from these two.

DIVISIBILITY BY NUMBERS ENDING IN 1, OF THE GENERAL FORM $10t+1$.

Proof is given for the subtraction rule.

Any whole number N may be expressed as $10t+u$.

Let R be the remainder after tu is subtracted from t. (See the table below for the general rule.) Then

$$\begin{aligned} N &= 10t+u \\ u &= N-10t \\ tu &= tN-10t^2 \\ R &= t-tu \\ &= t-tN+10t^2 \\ &= t(10t+1)-tN \end{aligned}$$

Since $10t+1$ will always divide the $t(10t+1)$ or R exactly, then R will have an integral quotient when divided by $10t+1$ only when $10t+1$ divides N exactly.

DIVISIBILITY BY NUMBERS ENDING IN 3, OF THE GENERAL FORM $10t+3$.

Proof is given for the addition rule in this case.

Any whole number may be expressed as $10t+u$.

Let S be the sum obtained by adding $(3t+1)u$ to t. (See the table below for the general rule.)

$$\begin{aligned} N &= 10t+u \\ u &= N-10t \\ (3t+1)u &= (3t+1)(N-10t) \\ S &= t+(3t+1)u \\ &= t+(3t+1)(N-10t) \\ &= t+(3t+1)N-30t^2-10t \\ &= (3t+1)N-3t(10t+3) \end{aligned}$$

Since $10t+3$ will always divide the $3t(10t+3)$ exactly and since $10t+3$ will not divide $3t+1$ exactly, then R will have an integral quotient when divided by $10t+3$ only when $10t+3$ divides N exactly.

Similar proofs may be developed for the tests of divisibility by numbers ending in 7 and 9.

RULES FOR TESTING DIVISIBILITY

Number	Multiple	Subtract from t in the number N to be tested	Add to t in the number N to be tested
11	11	$1u$	$10u$
21	21	$2u$	$19u$
31	31	$3u$	$28u$
.	.	.	.
.	.	.	.
.	.	.	.
$10t+1$	$10t+1$	tu	$(9t+1)u$
3	21	$2u$	$1u$
13	91	$9u$	$4u$
23	161	$16u$	$7u$
.	.	.	.
.	.	.	.
.	.	.	.
$10t+3$	$10(7t+2)+1$	$(7t+2)u$	$(3t+1)u$
7	21	$2u$	$5u$
17	51	$5u$	$12u$
27	81	$8u$	$19u$
.	.	.	.
.	.	.	.
.	.	.	.
$10t+7$	$10(3t+2)+1$	$(3t+2)u$	$(7t+5)u$
9	81	$8u$	$1u$
19	171	$17u$	$2u$
29	261	$26u$	$3u$
.	.	.	.
.	.	.	.
.	.	.	.
$10t+9$	$10(9t+8)+1$	$(9t+8)u$	$(t+1)u$

A Test Plus a Bonus

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MY FOURTH-GRADERS enjoy their weekly arithmetic test. I can't really claim that none of the children approaches it with anxiety, but the general atmosphere in our classroom on Friday morning is one of excitement and anticipation. The children voluntarily limit their discussion period so they can "get to the test" and have enough time to do a good job. Frequently when our appointment with the music teacher signals the end of the test there are sighs of satisfaction and murmurs of "That was fun!" along with the questions and comparisons that usually follow a test.

The test itself looks quite ordinary. Like most classroom teachers, I try to make it a fair measure of the children's accumulated skills and understandings as well as their very recent experiences. But since the testing routine has become so satisfying to the children and to me, I've tried to isolate the aspects of the test, beyond the usual criteria for a useful instrument to measure pupil progress, which make us enthusiastic. There are three such aspects, the last of which I shall describe most fully.

First, each Friday's test is tailor-made for a particular group at a particular time. The ditto sheets come to the children still smelling of duplicator fluid. (They are often held up to wrinkled little noses for appreciative sniffs.) The previous day's activities, as well as that morning's newscast, may be reflected in the problems.

Second, the children have participated, not only in the activities that are described in the problems, but also in the formulation of the test itself. The most appropriate of the children's oral discussion is used, as well as the best of accumulated written problems. Credit is given for ideas and for original stories by writing children's names in the margins of the test sheets.

Third, when a child has completed the test and has checked and proofread his papers, he may go pick up his "dessert"—a set of bonus questions designed to challenge the gifted, to stimulate interest, to encourage timid thinkers by demonstrating that old principles can be applied to new problems, to entertain, and (I admit) to use constructively and quietly the extra time available to the children who finish the test so much more quickly than others.

After several years of experimentation with the idea of a bonus, a routine procedure has evolved. And the children seem to enjoy a predictable test form when the content is challenging. The children know that there will be four bonus questions, that not more than one of the four will be "tricky," that if they arrive at three correct answers they "win," and that there is never any penalty connected with trying but not succeeding at this part of the test. Even the children who seldom succeed beg for a few minutes to try the bonus.

Because there is so much good material available, I often adapt what I find rather than create something original each time. (At the end of this article there is a list of suggested sources.) Each week the set of questions covers a variety of ideas, but from week to week there is a thread of continuity so that the questions not only become more difficult but are designed to develop increasing insight by following through an idea until it becomes a familiar approach to numbers and the children recognize it with confidence. The four questions are chosen from four different categories, and of course the list of categories keeps growing. Here are some of the kinds of bonus questions that have been successful in the fourth grade:

1. Progressions of increasing difficulty.
2. Crossnumber puzzles.

3. Magic squares, circles, etc.
4. Riddles involving numbers.
5. Construction puzzles.
6. Word puzzles (anagrams, etc.) using arithmetic words.
7. Timely messages in code, using Roman as well as Hindu-Arabic numbers. The day after it became certain that Hawaii would be a state, this was one of our bonus questions:

VIII—I—XXIII—I—IX—IX
XXIII—IX—XII—XII
II—V
XV—XXI—XVIII
VI—IX—VI—XX—IX—V—XX—VIII
XIX—XX—I—XX—V.

Hint:
I = *a*
II = *b*
III = *c*

8. Problems in which the knowledge of nursery rhymes is necessary. One of the children submitted this problem, which we used as a bonus question:

> Multiply the number of blackbirds baked in a pie by the number of bags of wool Baa Baa Black Sheep had, and then divide by the number of fiddlers called for by Old King Cole.

9. Problems based on recall of the past week's experiences.
10. Examples looking ahead to something that hasn't been taught yet: multiplying by a three-place number after the children are quite accustomed to multiplying by a two-place number, or dividing by a two- or three-place number when the class is familiar with the idea of division as repeated subtraction.
11. Questions involving an unusual use of numbers (circular counting on the clock, for example, which makes eleven plus two equal one).
12. Patterns developed in the main part of the test, to be identified in the bonus section. Sometimes the answers turn out to be a progression. Once a year the children are given a series of examples in which 37 is to be multiplied by 3, 6, 9, 12, 15, 18, 21, 24, and 27—which results in an exciting progression. However, I usually break such a pattern and ask the children not only to identify the pattern but also to find the flaw and correct it.
13. Experimenting with number relationships. The test may suggest a rule for determining which numbers are divisible by 3 or by 4 without remainder, for example, and the children try it out. Some children will be satisfied to test the rule itself, while others will attempt to find out why the rule works. Nine is a particularly interesting number to play with.
14. Finding missing numbers. The first question of this kind is usually a subtraction example:

$$\begin{array}{r} \$20.00 \\ -\,??.?? \\ \hline \$\ 5.02 \end{array}$$

Later, the children get a feeling of satisfaction from discovering that they can use their method of checking to find the missing dividend:

$$\begin{array}{r} 234 \\ 6\overline{)????} \end{array}$$

The list is not intended to be exhaustive. In choosing bonus questions it is important to keep the test interesting and varied, but merely selecting questions at random will not be consistently successful. At least part of the material each week should be related to what the children have been doing; the test must be just hard enough so that they will have to reach for it; and there should be enough continuity and consistency for the children to develop increasing insight and confidence.

Some classes have been quite indifferent to the idea of a bonus at the beginning of the year, but there are always a few children who find it challenging. Before long, there are requests for extra copies for parents and brothers and sisters, who reciprocate by sending in their favorite puzzles. The appropriate ones are put aside for Friday's tests; the others are discussed in class or discarded. The children are encouraged to collect us-

able items and to give the sources of what they submit. As soon as they realize that their contributions will be taken seriously and that they will be given credit for their ideas, they co-operate enthusiastically and interest mounts. Of course, the final selection is still the teacher's responsibility, and there is so much material readily available that she doesn't really have to depend on student research.

Testing can be satisfying (fun, sometimes) to children if the test is immediate and timely, if the children have been invited to participate by contributing material and their contributions are taken seriously, and if at the end of the test there is a challenge and an invitation to experiment and think.

There are so many lists of enrichment material that I need to append only a short, suggestive "starter" list:

Herbert F. Spitzer, *Practical Classroom Procedures for Enriching Arithmetic*. St. Louis: Webster Publishing Co., 1956.

"Enrichment Program for Arithmetic" Booklets. Evanston, Illinois: Row, Peterson & Co., 1956.

Various manuals for teachers.

Articles, reviews, and advertisements in THE ARITHMETIC TEACHER.

EDITOR'S NOTE. This testing is much more than routine evaluation of learning. It is in fact a very interesting device to stimulate interest and learning. The wide participation of pupils in the finding of material for test items and in working for "bonus points" should set a general tenor for the work in arithmetic. It is interesting to note the inclusion of items that call for thinking ahead of the normal class development. This type of testing lends itself to many different aspects of the whole area of mathematics.

Let's Prove It!

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IN THE PAST FEW YEARS educators have been acutely aware of the attempts being made to revise the mathematics curriculum in our schools. The work of the School Math Study Group, Maryland Project and the University of Illinois has produced some extensive work and experimentation to improve, not only, the content but the method as well. If you have not tried some of their ideas in your class may I suggest that you do. The following problems were enjoyed by both seventh and eighth grade students.

1. Show that: $A+(B+5)=C+(B+A)$.

$A+(B+C)$

$=(A+B)+C$ (associative law)

$(A+B)+C$

$=(B+A)+C$ (commutative law)

$(B+A)+C$

$=C+(B+A)$ (commutative law)

2. Show that: *The sum of any two odd numbers is an even number.* N and M are two natural numbers. They could be odd or they could be even. However, $2N$ and $2M$ are even numbers, any number that has 2 as a factor is even. $2N+1$ and $2M+1$ are odd numbers.

$2N+1+2M+1=2N+2M+1+1$

(commutative law)

$2N+2M+1+1=2N+2M+1$

$2N+2M+2=2(N+M+1)$

(distributive law)

$2(N+M+1)$ is even.

3. Show that: *The sum of any three odd numbers is an odd number.* N, M and P are natural numbers. $2N$, $2M$ and $2P$ are even numbers; $2N+1$, $2M+1$, and $2P+1$ are odd numbers.

$2N+1+2M+1+2P+1$

$=2N+2M+2P+1+1+1$

(commutative law)

$=2M+2N+2P+2+1$ (express 3 as $2+1$)

$=2(M+N+P+1)+1$ (distributive law)

$2(M+N+P+1)$ is even, therefore,

$2(M+N+P+1)+1$ is odd.

In teaching the distributive law do not pass over such examples as $x(4+3)$; $m(n+p)$; $y(y+3)$ or $x(2x^2+3x+2)$. It is equally important that the pupils be given the opportunity to distribute *out* as well as *in.* i.e. $4x+8y=4(x+2y)$. As a real test of understanding have them use the distributive law on A 1 (B * c) or even $A\cup(B\cap C)$. Your better pupils will enjoy distributing

$(a+b)(2a+2b)=(a+b)2a+(a+b)2b=$

$2a(a+b)+2b(a+b)$ (commutative law)

$=2a^2+2ab+2ab+2b^2$ (distributive law)

$=2a^2+4ab+2b^2$.

Be certain to tell them that this later type of work is usually found in the second semester of the 9th grade algebra class, then stand back and watch them distribute "till your heart's content."

An interesting problem is that of determining when a fraction is in lowest terms. *If a fraction can be reduced, then it will be reduced by one or more of the prime factors of the difference of the numerator and the denominator.*

Any rational number (fraction) may be expressed in the form

$$\frac{N}{N+M}.$$

$\frac{N}{N+M}$ is not in lowest terms.

Temporary premise.

If

$$\frac{N}{N+M}$$

is not in lowest terms, then N and $N+M$ have at least one factor in common.

$N=ka$. (N has at least two factors.)

By substitution we obtain

$$\frac{ka}{ka+M}.$$

If

$$\frac{ka}{ka+M}$$

can be reduced then it must be in the form

$$\frac{ka}{k(a+b)}$$

or

$$\frac{ka}{a(k+b)}.$$

Both of which can be reduced by one or more of the factors of the difference of the numerator and denominator.

Students of logic will recognize the "if . . . then . . . " statement as $p\rightarrow q$, which is logically equivalent $\sim q\rightarrow\sim p$, which is to say that if the prime factors of the difference will not reduce the fraction it cannot be reduced.

Now let us try this method and see if we can reduce 38/57. $57-38=19$. The prime factor(s) of 19 is 19. If our fraction can be reduced it will be reduced by 19. $38/57 =(2\times 19)/(3\times 19)=1\times\frac{2}{3}=\frac{2}{3}$.

EDITOR'S NOTE. Our better pupils in grades seven and eight delight in discovery and application of new principles and are not disturbed by the more general algebraic symbolism. They can establish the associative and commutative laws inductively and then proceed to apply them. This type of work will be new to many arithmetic teachers. It is not introduced merely because it is new but because it leads to a higher level of mathematical understanding and appreciation.

The Meaning of Two Times Two

G. T. Buckland
Appalachian State Teachers College, Boone, N. C.

For a large number of people, laymen, students, as well as teachers, two times two has always meant four. The reason for this seems to be "just because it has always been that way." Actually two times two has two other products equally as good as the common four. Sometimes the product could be 10 or 11. These answers are not to be confused with the numbers ten and eleven involved in our common base of operation, the decimal base.

Assuming that our product is to be four, we are faced with the reasoning behind such a decision. First, it might be stated that two times two equals four because it has been agreed that this was to be the product. It is, however, necessary to go back and investigate the various assumptions required in order to always come up with the same answer. These assumptions are four in number and are as follows:

1. Peano's postulates. These postulates, formulated by an Italian mathematician, are necessary for our number system to retain order and remain consistent.

(1) We must accept 1 to be a natural number.

(2) For each natural number there must also exist another number, unique in nature, which will be its successor.

(3) There cannot exist a natural number which will have 1 as its successor. This involves the idea of zero being considered more as a place holder than anything else.

(4) Should there be two natural numbers with the same successors, then it would mean that the two numbers were the same originally.

(5) If we have 1 and its successor and all the successors for each of the other natural numbers in turn, then we have the complete listing of all natural numbers.

2. Associative law of addition. This law states that one number can be added to the sum of two other numbers such as $a+(b+c)$ giving a sum equivalent to the sum of two numbers added to a third number such as $(a+b)+c$. The result being $a+(b+c)=(a+b)+c$.

3. Distributive law. Briefly, this law states that in the process of multiplying one number by the sum of two numbers that each number might be multiplied by the coefficient and the sum then taken as follows: $a(b+c)=ab+ac$.

4. Identity element. The identity element referred to is under the operation of multiplication. It has been defined as that number which when multiplied by any given number does not change the number. Therefore the identity element under multiplication is 1.

Now by following through with the four assumptions it can readily be shown that the idea of $2\times2=4$ begins to take on meaning. Logical reasoning combined with accepted facts tend to produce desired results in mathematics regardless of the complexity of the situations involved.

A very simplified resume of the operations involved in showing that two times two equals four might be shown as follows:

1 is a natural number	(Peano's postulate 1)
$1+1=2$	(Peano's postulate 2)
$2\times1=2$	(1 is the identity element)
$2\times2=2(1+1)$	(We just assumed that the $1+1=2$)
$2(1+1)=2+2$	(Distributive law)
$2+2=2+(1+1)$	(As of above)
$2+(1+1)=(2+1)+1$	(Associative law of addition)
Therefore $2+1=3$	(Peano's postulate 2)
Then $(2+1)+1=3+1$	
So $3+1=4$	(Peano's postulate 2)
Which makes $2\times2=4$	(The assumption justified)

There are other methods, some probably shorter, that might be used to obtain the same results. However, the importance of Peano's postulates and the idea of successors cannot be disregarded when work involving the natural numbers is concerned.

Although it has been shown that two times two equals four, it can also be shown that the possibility of two other answers, just as legitimate, can also exist.

The foregoing explanation, based upon the idea of a base of ten, can be extended to cover bases of three and four. Products of two times two in bases of five or greater bases would give the traditional answer of four. However, there is no answer for the same problem in base two for under these conditions it does not exist. In order to explain the procedure involved in multiplying in any and all bases it might be worthwhile to follow the procedure in base ten (decimal) first in order to see the similarity in all bases.

In the problem (in base 10)

$$\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array}$$

the procedure involved is identical in all bases. Generally it is said that $2\times 2=4$. The 4 is divided by the base as follows:

$$\begin{array}{r@{}l} & \;0 \\ \cline{2-2} 10) & \;4 \\ & \;0 \\ \cline{2-2} & \;4 \end{array}$$

giving a quotient of zero (which is to be carried) and a 4 as the remainder (which goes in the product) as follows:

$$\begin{array}{r} {}^{0}\;\;2 \\ \times 2 \\ \hline 4 \end{array}$$

adding gives

$$\begin{array}{r} {}^{0}\;\;2 \\ \times 2 \\ \hline 04 \end{array}$$

and naturally the zero to the left is dropped giving the answer

$$\begin{array}{r} 2 \\ \times 2 \\ \hline 4 \end{array}$$

In base four the same procedure is followed:

2×2 (base 4) gives 4 (thinking in base ten) this 4 is divided by the base as follows:

$$\begin{array}{r@{}l} & \;1 \\ \cline{2-2} 4) & \;4 \\ & \;4 \\ \cline{2-2} & \;0 \end{array}$$

giving a 1 to carry with the remainder zero going into the product:

$$\begin{array}{r} {}^{1}\;\;2 \\ \times 2 \\ \hline 0 \end{array}$$

(base 4). There being no number to combine with the 1, it is brought down in the product giving 10. Therefore 2×2 (base 4) $=10$. The 10 is not to be called ten or confused with the ten that it resembles in base ten. However, it is found to be the fourth number when counting takes place in base four as follows: 0, 1, 2, 3, 10, . . . , (the element 0 is not considered in the counting process).

If base three is to be considered it will be found to react in a way quite similar to the other two bases just explained. In base three:

2×2 (base 3) gives 4 (thinking in base ten). This 4 is divided by the base as follows:

$$\begin{array}{r@{}l} & \;1 \\ \cline{2-2} 3) & \;4 \\ & \;3 \\ \cline{2-2} & \;1 \end{array}$$

giving a 1 to carry with the remainder 1 going to the product:

$$\begin{array}{r} {}^{1}\;\;2 \\ \times 2 \\ \hline 1 \end{array}$$

(base 3). There being no number to combine with the 1, it is brought down in the product giving 11. Hence 2×2 (base 3) $=11$. Likewise the 11 is not to be called eleven or considered as eleven in base ten. In counting (base 3) it is found to be the fourth number as follows: 0, 1, 2, 10, 11, . . . (the element 0 is not considered in the counting process).

Arithmetic and the processes involved are not static, therefore different results can be obtained depending upon the point of view taken. The whole idea is manmade and is subject to changes, revisions, additions, and deletions. However, as long as our thinking follows some logical pattern and we accept the required assumptions our answers will always satisfy the conditions necessary for the solutions of our problems.

EDITOR'S NOTE. Many people tend to accept mathematical principles which are based upon definitions and assumptions as entities which are inherent in the structure of the universe when in fact they have been created by man in order to interpret and explain phenomena in this universe. It is a tribute to man's creation of mathematical systems that mankind continues to place such great faith in these systems. Most people will not be greatly concerned by postulates of mathematics, they will trust the mathematicians for competence in this area.

Editor's Note—Prospective Teachers

(*Continued from page 146*)

EDITOR'S NOTE. The author is justly disturbed about the effects of having arithmetic taught by people who have a deficient knowledge in this important subject. It is interesting to note that these people all have had a four-quarter-hour course in mathematics. Results of the test would probably have been considerably lower had this previous course not been taken, One might ask if an introductory college course in mathematics might not well concern itself with some of the aspects of plain useful arithmetic. Since there is a marked difference between those who had three years of high school mathematics and those with less, we might argue for requiring three years for college entrance. It may not be the amount of mathematics *per se* that is the factor but the elements of selectivity which distinguishes the one group from another. This selectivity involves several factors including level of ability and mental attitude toward mathematics. This same factor of selectivity may account for higher scores made by juniors and seniors and the additional element of normal college attrition. The author's findings are in harmony with other similar studies. One can hope that our future elementary school teachers might improve in two aspects: (a) their understanding of and ability to perform in arithmetic and (b) their attitude including the lack of fear toward the subject. It may be noted that the test covered computation and written problems and did not include the measurement of understanding of concepts and principles except as these become a part of other items.

Editor's Note—Content-Methods Course

(*Continued from page 140*)

A COMPARISON OF THREE TYPES OF ARITHMETIC COURSES IN LIGHT OF EIGHT MAIN AIMS THEORETICALLY SET UP

The General Aims and Qualities	Subject Matter Course	Methods Course	Professionalized Subject Matter
1. Mastery of ordinary arithmetic	A	D	C
2. Background of material	C−	D	C+
3. Educational principles	D	C+	C−
4. Techniques of teaching	D	A	C−
5. Inherent philosophy	C−	C−	C
6. Materials of instruction	D	C	C−
7. Child-education point of view	D	C+	C−
8. Researches and experiments	D	C	C

The chart should be interpreted according to the key: A=overemphasis, C=adequate, and D=deficient. Item one is read across as: in teaching mastery of ordinary arithmetic the Subject Matter Course gives overemphasis, the Methods Course is deficient, and the Professionalized Subject Matter Course is adequate.

38th Annual Meeting of the National Council of Teachers of Mathematics Hotel Statler, Buffalo, New York April 21-23, 1960

PROGRAMS FOR ELEMENTARY SCHOOL TEACHERS

THURSDAY AFTERNOON—APRIL 21

1:30 P.M.–3:00 P.M. *Elementary School Section* Maple Leaf Room
Presiding: BEN A. SUELTZ, State University College of Education, Cortland, N. Y.
HOW TO IMPROVE THE ELEMENTARY-SCHOOL MATHEMATICS CURRICULUM?
J. Fred Weaver, Boston University School of Education, Boston, Mass.
SET, SCALE, SYMBOL
Robert L. Swain, Rutgers, The State University, New Brunswick, N. J.

3:15 P.M.–4:45 P.M. *Elementary School Laboratory* Maple Leaf Room
Presiding: RUTH S. MOLL, Green Acres Elementary School, Kenmore, N. Y.
A LABORATORY OF IDEAS FOR THE ELEMENTARY SCHOOL
Ann C. Peters, Keene Teachers College, Keene, B. H.

3:15 P.M.–4:45 P.M. *The Twenty-Fifth Yearbook* Los Angeles-Washington Room

Arithmetic in Today's Culture

Presiding: JOYCE BENBROOK, University of Houston, Houston, Tex.
THE POINT OF VIEW OF THE TWENTY-FIFTH YEARBOOK
Foster E. Grossnickle, Jersey City College, Jersey City, N. J.

3:15 P.M.–4:45 P.M. *Research Section* Empire State Room
Presiding: JOHN J. KINSELLA, New York University, New York, N. Y.
THE EFFECTIVENESS OF MODERN MATHEMATICS IN THE PREPARATION OF ELEMENTARY SCHOOL TEACHERS
L. Coleman Knight, Muskingum College, New Concord, Ohio
THOUGHT PROCESSES OF SIXTH GRADE PUPILS WHILE SOLVING VERBAL PROBLEMS IN ARITHMETIC
Clyde G. Corle, The Pennsylvania State Univ., University Park, Pa.
A COMPARISON OF TWO METHODS OF TEACHING ALGEBRA IN THE NINTH GRADE
Nicholas Kushta, Chicago Public Schools, Chicago, Ill.

FRIDAY, APRIL 22

10:15 A.M.–12:00 NOON *Elementary School Section* Georgian Room
Presiding: JOSEPHINE H. MAGNIFICO, Longwood College, Charlottesville, Va.
DEPTH IN ARITHMETIC LEARNING, THE WHY AND HOW
Charlotte W. Junge, Wayne State University, Detroit, Mich.
TEACHING THE LANGUAGE OF PER CENT
John L. Marks, San Jose State College, San Jose, Calif.

10:15 A.M.–12:00 NOON *Teacher Education Section* Empire Room
Presiding: RONALD C. WELCH, Indiana University, Bloomington, Ind.
THE SUBSET: ELEMENTARY TEACHERS; A TRUE SUBSET?
E. W. Hamilton, Iowa State Teachers College, Cedar Falls, Iowa
Panel: Ann C. Peters, Keene Teachers College, Keene, N. H., Moderator
Clarence Ethel Hardgrove, Northern Illinois Univ., DeKalb, Ill.
Harold E. Moser, State Teachers College, Towson, Md.
M. Isabelle Savides, Meadowbrook Junior High School, Newton, Mass.
Alfred B. Wilcox, Amherst College, Amherst, Mass.

2:00 P.M.–3:30 P.M. *Elementary School Section*
Presiding: EDWINA DEANS, Arlington Public Schools, Arlington, Va.
DEMONSTRATION CLASS OF 5th–6th GRADE CHILDREN TO ILLUSTRATE CHILDREN AND SYMBOLISM
David A. Page, Director, University of Illinois Arithmetic Project, Urbana, Ill.

SATURDAY, APRIL 23

10:15 A.M.–11:45 A.M. *Elementary School Section* — Maple Leaf Room
Presiding: RUTH H. TUTTLE, Denver Public School, Denver, Colo.
DEFINITIONS IN ARITHMETIC
William B. Higgins, Ball State Teachers College, Muncie, Ind.
A NEW LOOK AT CONTENT AND ITS PLACEMENT IN ELEMENTARY MATHEMATICS
Herbert Hannon, Western Michigan University, Kalamazoo, Mich.

10:15 A.M.–11:45 A.M. *Lecture Section* — St. Louis-Boston-Detroit Room
Presiding: JOHN L. MARKS, San Jose State College, San Jose, Calif.
FRACTIONS, RATIOS, AND PER CENTS
Henry VanEngen, University of Wisconsin, Madison, Wis.

2:15 P.M.–3:45 P.M. *Elementary-Junior High School Section* — Georgian Room
Presiding: BEULA M. WILSON, Euclid Public Schools, Euclid, Ohio
ALGEBRA IN GRADES FIVE THROUGH EIGHT
W. Warwick Sawyer, Wesleyan University, Middletown, Conn.

2:15 P.M.–3:45 P.M. *Section on Individual Differences* — Maple Leaf Room
Presiding: VINCENT J. GLENNON, Syracuse University, Syracuse, N. Y.
OBTAINING GREATER INDIVIDUAL DIFFERENCES
Robert E. Pingry, University of Illinois, Urbana, Ill.
GUIDANCE AND COUNSELING IN ARITHMETIC
Irene Sable, Detroit Public Schools, Detroit, Mich.

A complete program of all meetings scheduled for the Annual Meeting of the National Council of Teachers of Mathematics will be mailed to all members in advance. Now is the time to make preliminary plans to attend and bring a friend.

Nebraska Meeting

The annual meeting of the Nebraska Section of The National Council of Teachers of Mathematics will be held on April 30, 1960 at Lincoln in the Public Schools Administration Building auditorium at 720 S. 22nd street. The meetings will begin at 9:00 A.M. with Dr. Monte S. Norton presiding. The Nebraska section has mimeographed materials on Number Systems and on The Binary System for distribution to teachers in the state.

Math Olympiads at Menlo Park, California

After hearing Professor Jerzy Gorski of Lodz, Poland lecture to an in-service institute and describe the Olympiads (contests) conducted in Poland, the institute at Menlo Park scheduled three Math Olympiads for grades five and six. The third and final in the series will be held in may and this will serve as "Play-offs" for winners in the first two. Teachers who are interested in more details may write to Mr. Lloyd V. Rogers, 125 Oak Court, Menlo Park, California.

Zero

Zero is a nothing
No matter how you use it;
A number never can be changed
When zero's added to it.
Can you divide by zero?
It simply can't be done.
The smallest number you can use
Would have to be one.
Zero is a place holder,
At that it's very good.
Zero is important
If you use it as you should.

CHERYL ODGERS
Grade 7B
Grosse Pointe, Mich.

Book Reviews

Mathematics, First Course, John A. Brown, Bona Lunn Gordey and Dorothy Sward with John R. Mayor, Consultant and General Editor (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1960) cloth, 323 pp., $3.40.

Mathematics, Second Course, John A. Brown, Bona Lunn Gordey and Dorothy Sward with John R. Mayor, Consultant and General Editor (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1960) cloth, 365 pp., $3.64.

These texts represent a praiseworthy effort by the authors to bring topics in modern mathematics into the junior high school mathematics program. They reflect the trend in content and method as proposed in the publications of the National Council of Teachers of Mathematics and similar groups. Since the approach and concepts in the texts are quite sophisticated at times, the teachers using these books should have had considerable rigorous training in modern mathematics, and only the students who have a good foundation in mathematics and a degree of mathematical maturity will be successful throughout the program. In the reviewers' opinion, this program is not for the weak student or for the teacher with limited preparation.

It is unfortunate that many typographical errors and examples of poor grammatical form have not been corrected by the proofreaders or the authors. For example, in the First Course, on page 36, an angle *ABC* is labeled *AAC* in the accompanying figure; on page 249, the graph is not completed; on page 252, a bar graph has no title; and on pages 269 to 272, the coordinate grids have been omitted. There are many other errors of this type. Poor sentence form is found on page 26 of the First Course, and on page 29 of the Second Course. Many figures and the corresponding text materials are on separate pages, necessitating the turning of the page to view the figure as the material is being read.

Beginning each book is a section on number concepts which will be new and interesting to the students. In general, these sections are well written, and seemed refreshing to the reviewers. The development of positional notation by means of bases other than ten is a new look at something old and familiar. The authors carefully distinguish between number and numeral. However, they ask several questions about digits before adequately defining or enumerating them. The study of number is extended to include directed or signed numbers and ordered number pairs.

The reviewers noted that some definitions were inaccurate, incomplete, or missing. On page 185 of the First Course, the authors give this problem: "Susan correctly spelled 80 words out of 100 words. Her score in per cent was 80/100. Write this number, using the % symbol." Teachers will agree that 80/100 is not a per cent. On page 102 of the First Course and on page 76 of the Second Course, factors of a number are defined as pairs of divisors of the number whose product is the number, but in the illustrative examples, the authors call all of the divisors of the number factors. Perhaps these divisors should be grouped so that their products are equal to the number. The definition of factors is too restrictive. Although "integers" are not defined in the First Course, the authors see fit to ask the student to recognize and to work with them. We feel that students may not be familiar with this term as they were probably taught to use "whole numbers."

In working with approximate numbers, the authors have violated the procedures generally accepted by the authorities in this field. There are examples of "ragged" decimals, which many others say have no purpose and may be defeating the purpose of teaching decimals. The procedure for

rounding numbers varies in the two books. In the First Course, the procedure follows the older method of adding one to the preceding digit if the discarded digit is five. In the Second Course, the authors change to a so-called statistical procedure, but this method is not the usual method as found on page 316 of the Twenty-second Yearbook of the National Council of Teachers of Mathematics.

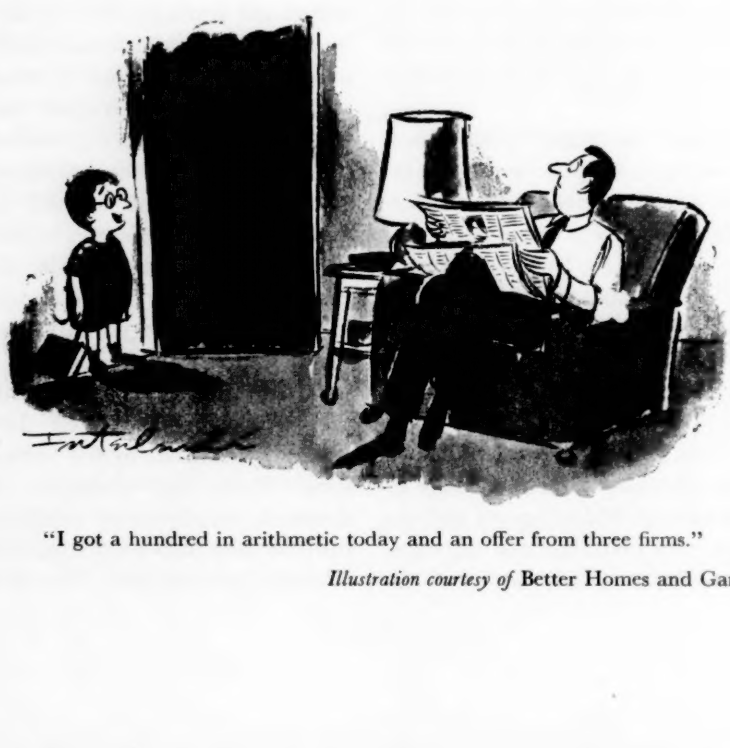
The photographs which accompany the texts are very attractive. However, the pictures of the Brussels World's Fair, representing fine examples of geometry in architecture, are not connected with the text in any manner. The photograph on page 155 of the First Course is not what is purports to be. A better picture could have been made in any super-market in the country.

The exercises are ample and varied; the Quick Quizzes provide a good means of building skills. Most of the exercises have problems of graded difficulty, but some of the problems are not realistic. On page 83 of the First Course is a problem concerning orders for grains measured to the nearest quarter pound. This degree of precision may cause snickers from some of the students who have a rural background. On page 220 of the Second Course, a bushel of corn is said to contain $2\frac{1}{2}$ cubic feet and a bushel of grain is said to contain $1\frac{1}{4}$ cubic feet. This seems to be inconsistent since a bushel is a measure of volume and no explanation is given.

The topics in probability and statistics found in these books are welcome additions, and provide some new applications of arithmetic at this level. The reviewers question the depth to which the authors have gone in the discussion of combinations and permutations. The trial use of these materials will determine the extent to which junior high school students can profitably handle these concepts.

Although the reviewers have cited many examples of errors and inconsistencies, these can be corrected easily. However, this should not be the responsibility of the classroom teachers, but of the authors. When these corrections have been made, the texts will represent a significant contribution to mathematics education in the junior high school. These books may well be the prototype of the future texts for this level.

JOHN C. BRYAN
ROBERT VANDAM
State University College of Education, Cortland, New York



"I got a hundred in arithmetic today and an offer from three firms."

Illustration courtesy of Better Homes and Gardens magazine